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Dr. Loyd A. Jones

IVES MEDALIST FOR 1943

AT its Pittsburgh meeting in October, 1943, the Optical Society of America awarded the Frederic Ives Medal to Dr. Loyd Ancile Jones, Chief Physicist of the Kodak Research Laboratories of the Eastman Kodak Company.

This medal is awarded biennially for distinguished work in optics, and its past recipients have been as follows:

In 1929, Edward Leamington Nichols;
In 1931, Theodore Lyman;
In 1933, Robert Williams Wood;
In 1935, George Ellery Hale;
In 1937, Herbert Eugene Ives;
In 1939, August Herman Pfund;
In 1941, Selig Hecht.

The presentation of the medal to Dr. Jones took place in the Lecture Room of the Mellon Institute, and was followed by an address by Dr. Jones.

Presenting Dr. Jones, the Chairman of the Committee appointed to nominate a candidate for the award spoke as follows:

"I was very much pleased when President Worthing asked me to be partly responsible for the selection of the one to receive this medal at this time. The first thing I did was to go over the list of men who had been selected for this honor in the past and study their attainments. Any assignment such as this carries with it the duty of selecting a candidate who will be of the same high standard as the men who have been given this honor in the past. A glance at the list of men who have been so honored will convince anyone that this requires some care.

"The first idea of the Committee* was that we must choose as our candidate someone who could take his place with this distinguished group. We did not want future committees to find an absorption band for the year 1943. Rather, we preferred committees which follow us to find a very strong emission band at this point, a band that would shine even in the above list of strong emission bands.

"Our selection, Loyd Ancile Jones, was born at York, Nebraska, on April 12, 1884, where he attended public schools and graduated from high school in 1903. He attended the University of Nebraska and received his B.S. degree in electrical engineering in 1908 and his M.S. in 1910. In 1933, he was awarded the honorary degree of Doctor of Science by the University of Rochester.

"From 1908 to 1910 he was Assistant in the Physics Department of the University of Nebraska, leaving there to join the Bureau of Standards staff as Assistant Physicist. In 1912, he joined the Kodak Research Laboratories as Assistant Physicist and has been Chief Physicist there since 1916.

"During the first World War Dr. Jones was a lieutenant in the United States Naval Reserve Force in charge of camouflage.

"Dr. Jones was President of our Society in 1930–31, and was President of the Society of

* H. K. Hartline, University of Pennsylvania Hospital, Philadelphia, Pennsylvania; Parry Moon, Massachusetts Institute of Technology, Cambridge, Massachusetts; S. H. Newhall, Johns Hopkins University, Baltimore, Maryland; R. Kingslake, Eastman Kodak Company, Rochester, New York; W. E. Forsythe, General Electric Company, Cleveland, Ohio, Chairman.

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Motion Picture Engineers, 1923–26. He has also served on many committees of each of these, as well as other scientific societies. He was a member of the Editorial Board of the Society of Motion Picture Engineers and did an enormous amount of work in getting their journal started. He is a member of the Committee on Nomenclature and Standards of the Illuminating Engineers Society, where he has been helpful in formulating the definitions of his field. He has been a member of the Optical Society Committee on Colorimetry for many years, and is now the Chairman of this group. This Committee has been working on a report which has finally taken on the form of a book on this subject. Arrangements have already been made for the publication of this book, and it would have been published before this except for the present disturbance.

"Along with his duties of directing and supervising the work of his associates in his department at the Research Laboratory of the Eastman Kodak Company, Dr. Jones has been actively engaged in his own researches in the fields of photometry, physical optics, illumination, colorimetry, physics of photography, visual sensitometry, and motion picture engineering. His outstanding achievements are in the field of photographic science, in recognition of which he has received several awards.

"Some ten years ago, Dr. Jones published a series of papers in the *Journal of the Society of Motion Picture Engineers* that have been collected to form a pamphlet called *Photographic Sensitometry*. This pamphlet has been of very great use to those who are interested in this subject.

"I had occasion some years ago to collect together material for a book on the general subject of 'Measurement of radiant energy.' This book was a collection of chapters written by men who were selected because they are experts in the particular fields covered by their writing. For this book, Dr. Jones wrote a chapter on 'Measurement of radiant energy with photographic materials.' This chapter covers some forty pages, and I think is authority in this field. If anyone has an occasion to measure radiation by photographic means, he will find a good guide in this chapter.

"Dr. Jones has already received a number of honors for his scientific work. In 1935, as co-author of a paper entitled 'Reciprocity law failure in photographic exposure,' he received the Journal Award of the Society of Motion Picture Engineers. That same year he was awarded a medal by the Royal Photographic Society of Great Britain for Motion Photomicrography of Crystals. (The crystal growth of certain organic compounds was photographed by polarized light on Kodachrome 16-mm motion picture film.) In 1939, Dr. Jones received the Progress Medal of the Society of Motion Picture Engineers 'in recognition of the outstanding character of his scientific research in the field of photography, with particular reference to his investigations of sensitometric procedure; his studies of photographic processes; his contribution to precision in photographic terminology; and his determinations of the criteria of pictorial excellence achieved by photographic processes.' In 1940, he received the Modern Pioneer Award of the National Association of Manufacturers Associated Industries for 'patented contributions to the physics of photography.'

"The results of his many extensive investigations in the field of physics of photography and allied fields of physics will be found in some eighty-odd published communications and reports. These papers include such subjects as the discharge of electricity through gases, photometry, reflectance standards, light sources for various purposes, and also many papers on color and its application, papers on photographic units of intensity, and a series of papers on photography, the photographic processes, photographic sensitometry, standards, and nomenclature.

"Dr. Jones is a leading world authority on two subjects, colorimetry and photographic sensitometry, and in each he has made many fundamental and important contributions. In colorimetry, the forthcoming report will show something of the extent of his activities in this field and the extent of his work on sensitometric procedures; methods for determining and specifying the sensitivity or speed of photographic emulsions based on subjective as well as objective considerations; and the development of the theory of tone reproduction, upon which depends a truthful tonal representation of the object photographed, is shown by the many published papers on this subject.

"Dr. Jones has already been honored by this Society for another achievement. In 1926, he received an award from the Association of Scientific Apparatus Makers of the United States of America for the best instrument paper offered for publication in the *Journal of the Optical Society of America and The Review of Scientific Instruments*. His paper was on 'A new method of photographic spectrophotometry.'

"As representative of the Frederic Ives Medal Committee, I presented Dr. Loyd A. Jones' name to the Board of Directors of the Optical Society as their selection, and he was elected by the directors to receive this honor. I wish, therefore, Mr. President, to present to you Dr. Loyd A. Jones to receive the Frederic Ives Medal from your hands."

President Worthing then presented the Frederic Ives medal with the following words: "Loyd Ancile Jones, in recognition of outstanding achievements in photographic research with special reference to sensitometric processes, photographic print quality, and motion photomicrography of crystals, of leadership in photographic terminology, and of leadership and supervision in the preparation of an excellent and comprehensive report on colorimetry, I herewith, in accord with the award made by the Optical Society of America, present to you its Frederic Ives Medal for 1943 for distinguished work in optics."

On presentation of the medal, Dr. Jones replied as follows:

"Even under normal conditions of emotional tranquillity I am not endowed with the gift of eloquence. In this situation when my emotions are stirred to their very depths I find myself even less capable of expressing the many thoughts which crowd my mind in disorderly array, in anything but a homely and unadorned style. One might suppose that after having listened to the eloquent remarks of Dr. Forsythe I would be so elevated in spirit as to speak in inspired phraseology. His panegyric, in which he ascribed to me a distinction quite out of proportion to my modest accomplishments, seems to have had exactly the opposite effect and to have driven from my mind those things which I had planned to say to you.

"I wish, however, first of all to express my deep and heartfelt gratitude for this honor which has been bestowed upon me this evening. My thanks and appreciation I extend to the award committee for its action, to the Board of Directors for affirming that committee's recommendation, and to the Optical Society of America which through its duly constituted officers and representatives has given to me the highest honor in its power to bestow. This material symbol which is emblematic of the honor which has been conferred upon me I shall cherish as one of my most precious possessions.

"I should like to make a few comments concerning this medal which President Worthing a few moments ago placed in my hands. It possesses a remarkable degree of beauty both intrinsically and extrinsically. Objectively, it is a thing of beauty because of the simplicity and proportion inherent in its design. But its greatest beauty lies in its extrinsic characteristics. First of all, it is symbolic of filial affection, one of the most admirable of human emotions. Thus it represents the love and appreciation of a son for his father who accomplished so much in scientific endeavor during his life. The design on the reverse face also symbolizes beauty. It represents the separation of white light into its chromatic components, thus reminding us of the rainbow, one of nature's loveliest phenomena, and, by allowing the imagination to wander farther afield, it is suggestive of all those myriad hues with which nature adorns herself, the multitudinous colors without which this world would be indeed a drab and dreary place.

"It seems fitting now that I should pay my respects to the little group of Frederic Ives Medalists which this evening I am privileged to join. When I read their names and contemplate the greatness of their individual and collective accomplishments, I feel very humble and very small indeed in mental stature. I wonder whether I am worthy to join this illustrious company. I only hope that my coming into this honored group will not dim appreciably the brilliance of the constellation. I cannot help but feel that my own luster, such as it may be, can only be enhanced by the reflected glory of my fellow medalists. To them I wish to extend greetings and to express my deepest satisfaction in being associated with them. I am not unmindful of the fact that some of them no longer are with us in the flesh. But I am sure that they are still here by

virtue of the things which they accomplished and for which you have honored them.

"Let me now congratulate the members of the Optical Society of America, individually and collectively. You have much of which to be proud because you are members of a very important group in our modern civilization. This group, though numerically small (containing at most a few hundred thousand members in the whole world), is of the greatest importance to our present and future culture. Our modern way of life differs from that of previous cultures largely because of the contributions which have been made by scientific workers. Scientific knowledge and its application is a relatively new thing in the evolution of mankind. It began effectively about three centuries ago, but even in this relatively short time it has profoundly influenced our daily lives and contributed much to our well-being. Dr. A. D. Little refers to this group as 'The Fifth Estate' and defines its composition as follows: 'The fifth estate is composed of those with the simplicity to wonder, the ability to question, the power to generalize, the capacity to apply. It is, in short, the company of thinkers, workers, expounders, and practitioners upon whom the world is absolutely dependent for the preservation and advancement of that organized knowledge which we call science.'

"In this year of our Lord 1943 the responsibility of the fifth estate has become even greater. The world depends largely on you not only for 'the preservation and advancement of that organized knowledge which we call science,' but also for the preservation of our very civilization, our culture, and our way of life.

"Finally in closing I must acknowledge my obligation to my co-workers in the Kodak Research Laboratories. What I have accomplished could not have been achieved without their whole-hearted cooperation and efficient assistance. I feel that this award you have given me this evening is an honor bestowed also upon my associates."

LIST OF SCIENTIFIC PUBLICATIONS OF L. A. JONES

(1) "Effect on the cathode-fall in gases produced by the evolution of gases from the cathode," Phys. Rev. **32**, 328–340 (March, 1911).

(2) "Transmission and reflection photometer for small areas" (with P. G. Nutting), J. Wash. Acad. Sci. **4**, 313–315 (June 19, 1914), Kodak Research Lab. Comm. No. 15.

(3) "Reflecting power standards" (with P. G. Nutting and F. A. Elliot), Trans. Am. Illum. Eng. Soc. **9**, 593–597 (1914).

(4) "Color of illuminants," Trans. I.E.S. **9**, 687–709 (1914).

(5) "A new standard light source," Trans. I.E.S. **9**, 716–733 (1914); Am. Gas Light J. **101**, 251–253 (1914).

(6) "Sensitometry of photographic papers" (with P. G. Nutting and C. E. K. Mees), Phot. J. **54**, 342–359 (Dec. 1914).

(7) "Relative photographic and visual efficiency of illuminants" (with M. B. Hodgson and K. Huse), J. Frank. Inst. **180**, 484–487 (Oct. 1915).

(8) "Fundamental scale of pure hue, and retinal sensibility to hue differences," J. Opt. Soc. Am. **1**, 63 (1917); J. Frank. Inst. **183**, 500–508 (April, 1917), Kodak Research Lab. Comm. No. 44 Abs.

(9) "Spectral selectivity of photographic deposits" (with R. B. Wilsey), Kodak Research Lab. Comm. No. 57, Phot. J. **58**, 70–81 (Feb. 1918); J. Frank. Inst. **185**, 231–267 (Feb. 1918).

(10) "An instrument for the measurement of the visibility of objects," Phil. Mag. **30**, 96–134 (Jan. 1920), Kodak Research Lab. Comm. No. 79.

(11) "A new non-intermittent sensitometer," J. Frank. Inst. **189**, 303–329 (March, 1920); Phot. J. **60**, 80–100 (March, 1920).

(12) "Photographic rendering of tone values," J. Frank. Inst. **189**, 469–484 (April, 1920).

(13) "Graininess in photographic deposits" (with N. Deisch), J. Frank. Inst. **190**, 657–683 (1920).

(14) "Subtractive colorimeter," J. Opt. Soc. Am. **4**, 420–431 (Nov. 1920).

(15) "Improved methods of illuminating motion picture theaters," Trans. I.E.S. **15**, 645–677 (Dec. 1920); Elec. Rev. **77**, 757–766 (Nov. 13, 1920).

(16) "Sensitometric study of the reduction of developing-out papers" (with C. E. Fawkes), J. Frank. Inst. **191**, 503–516 (April, 1921).

(17) "Absorption of light by toned and tinted motion picture film" (with C. W. Gibbs), J. Frank. Inst. **192**, 676–677 (Nov. 1921).

(18) "New sensitometer for determination of exposure in positive printing" (with J. I. Crabtree), J. Soc. Mot. Pict. Eng. No. 15, 89–100 (1922).

(19) "Use of artificial illuminants in cinema studios," Illum. Eng. **15**, 271–275 (Sept.-Oct. 1922).

(20) "Gloss characteristics of photographic papers" (with M. F. Fillius), J. Frank. Inst. **193**, 835–836 (June, 1922).

(21) "The graininess of photographic materials used in the motion picture industry" (with A. C. Hardy), J. Soc. Mot. Pict. Eng. **14**, 107–119 (1922); Am. Cinemat. **3**, 7, 16.

(22) "A spectral energy sensitivity of photographic materials" (with A. L. Schoen), Kodak Research Lab. Comm. No. 166, J. Opt. Soc. Am. and Rev. Sci. Inst. **7**, 213–217 (March, 1923).

(23) "An instrument for the measurement of high photographic densities," Kodak Research Lab. Comm. No. 160, J. Opt. Soc. Am. and Rev. Sci. Inst. **7**, 231–242 (March, 1923).

(24) "A special sensitometer for the study of the photographic reciprocity law," Kodak Research Lab. Comm. No. 161, J. Opt. Soc. Am. and Rev. Sci. Inst. **7**, 305–319 (April, 1923).

(25) "Electrical and illuminating equipment of the Eastman Theatre and School of Music" (with F. A. Malt), Trans. A.I.E.E. **42**, 569 (June, 1923).
(26) "Recent progress in the sensitometry of photographic materials," Trans. Faraday Soc. **19**, 367–378 (1923).
(27) "On the relation between time and intensity in photographic exposure" (with E. Huse), Kodak Research Lab. Comm. No. 193, J. Opt. Soc. Am. and Rev. Sci. Inst. **7**, 1079–1113 (Dec. 1923).
(28) "Color measurements," Am. Dyestuff Rept. **13**, 121–131 (1924).
(29) "A new method for photographic spectrophotometry," Kodak Research Lab. Comm. No. 221, J. Opt. Soc. Am. and Rev. Sci. Inst. **10**, 561–572 (May, 1925).
(30) "Incandescent tungsten lamp installation for illuminating color motion picture studio," J. Frank. Inst. **200**, 675–676 (Nov. 1925).
(31) "Annular-oblique illumination for the microscopy of surface textiles, Paper Trade J. **83**, No. 16, 56–65 (1926).
(32) "Spectral distribution of sensitivity of photographic materials" (with O. Sandvik), Kodak Research Lab. Comm. No. 256, J. Opt. Soc. Am. and Rev. Sci. Inst. **12**, 401–416 (April, 1926).
(33) "Unit of photographic intensity" (with others), J. Opt. Soc. Am. **12**, 567–586 (1926).
(34) "Retinal sensitivity to saturation differences" (with E. M. Lowry), Kodak Research Lab. Comm. No. 254, J. Opt. Soc. Am. and Rev. Sci. Inst. **13**, 25–34 (July, 1926).
(35) "Contrast of photographic printing paper," Kodak Research Lab. Comm. No. 264, J. Frank. Inst. **202**, 177–204 (Aug.); 469–516 (Oct.); 589–625 (Nov. 1926); **203**, 111–156 (Jan. 1927); **204**, 41–89 (July 1927).
(36) "Relation between time and intensity in photographic exposure" (with V. C. Hall and R. M. Briggs), J. Opt. Soc. Am. **14**, 223–232 (1927).
(37) "Photographic spectrophotometry in the ultraviolet," Kodak Research Lab. Comm. No. 314, Nat. Research Council Bull. **61**, 109–123 (July, 1927).
(38) "Light filters, their characteristics and applications in photography," J. Soc. Mot. Pict. Eng. **11**, 135–181 (Aug. 1927)
(39) "Photographic characteristics of motion picture studio light sources" (with M. E. Russell), J. Soc. Mot. Pict. Eng. **12**, 427–447 (April, 1928).
(40) "Light filters for the isolation of narrow spectral regions," Kodak Research Lab. Comm. No. 335, J. Opt. Soc. Am. and Rev. Sci. Inst. **16**, 259–271 (April, 1928).
(41) "Relation between time and intensity in photographic exposure" (with V. C. Hall), Proc. 7th Intern. Cong. Phot. **1928**, 115–128 (1929).
(42) "Expression of (photographic) plate speed in terms of minimum useful gradient" (with M. E. Russell), Proc. 7th Intern. Cong. Phot. **1928**, 130–143 (1929).
(43) "High intensity time-scale sensitometer" (with G. A. Chambers), Proc. 7th Intern. Cong. Phot. (Heffer & Sons, Ltd., Cambridge), 206–216 (1929), Kodak Research Lab. Comm. No. 353.
(44) "Systematic nomenclature in photographic sensitometry," Kodak Research Lab. Comm. No. 350, Proc. 7th Intern. Cong. Phot. (Heffer & Sons, Ltd., Cambridge), 265–272 (1929).
(45) "Tinted films for sound positives," J. Soc. Mot. Pict. Eng. **13**, 199–226 (May, 1929).
(46) "Isolation of narrow spectral regions by selective absorption," J. Nutrition **2**, 111–123 (1929).
(47) "Rectangle proportions in pictorial composition," J. Soc. Mot. Pict. Eng. **14**, 32–49 (Jan. 1930).
(48) "Light filter for the isolation of narrow spectral regions," Phot. J. **70**, 337–346 (July, 1930), Kodak Research Lab. Comm. No. 335.
(49) "On the theory of tone reproduction, with graphic method for solution of problems," J. Soc. Mot. Pict. Eng. **16**, 568, 99 (May, 1931); J. Frank. Inst. **190**, 39–90 (July, 1920).
(50) "Unit of photographic intensity and the present status of its international standardization," J. Opt. Soc. Am. **21**, 361–368 (June, 1931).
(51) "Motion picture laboratory sensitometer," J. Soc. Mot. Pict. Eng. **17**, 536–559 (Oct. 1931).
(52) "Photographic sensitometry," J. Soc. Mot. Pict. Eng. **18**, 54–89, 324–355 (1932); **17**, 491–535, 695–742 (1931).
(53) "What can science do for practical determination of color standards for printing," Third Conf. of Tech. Experts in Printing Ind., Proc. and Papers Mtg. (March 14–15, 1932).
(54) "Precise measurement of filter factors and photographic reflecting powers" (with J. W. McFarlane), J. Soc. Mot. Pict. Eng. **19**, 361–390 (1932).
(55) "Historical summary of standardization in the Society of Motion Picture Engineers," J. Soc. Mot. Pict. Eng. **21**, 280–293 (1933).
(56) "Reciprocity law in photographic exposure" (with J. H. Webb), J. Soc. Mot. Pict. Eng. **23**, 142–158 (1934).
(57) "Minimum useful gradient as a criterion of photographic speed" (with M. E. Russell), Kodak Research Lab. Comm. No. 556, J. Opt. Soc. Am. **25**, 396–411 (Dec. 1935); Phot. J. **75**, 657–675 (Dec. 1935).
(58) "Developing machine for sensitometric work" (with M. E. Russell and H. R. Beacham), J. Soc. Mot. Pict. Eng. **28**, 73–98 (1937).
(59) "Nomenclature for photometry and colorimetry," J. Opt. Soc. Am. **27**, 207–213 (June, 1937).
(60) "Evaluation of negative film speeds in terms of print quality," Kodak Research Lab. Comm. No. 683, Phot. Ind. **37**, 163–176 (Feb. 15, 1939); J. Frank. Inst. **227**, 297–354 (March, 1939); 497–544 (April, 1939).
(61) "Isolation of desired qualities of radiant energy for biological experimentation" (with C. Tuttle), J. Nutrition **18**, 611–618 (1939).
(62) "Sensitometry of photographic papers" (with C. A. Morrison), Kodak Research Lab. Comm. No. 737, J. Frank. Inst. **228**, 445–470 (Oct. 1939); 605–622 (Nov. 1939); 755–769 (Dec. 1939).
(63) "Sensitometric criteria for effective speeds of negative film" (with C. N. Nelson), Kodak Research Lab. Comm. No. 748, J. Opt. Soc. Am. **30**, 93–109 (March, 1940); Phot. J. **80**, 152–168 (April, 1940).
(64) "The brightness scale of exterior scenes and computation of correct photographic exposure" (with H. R. Condit), Kodak Research Lab. Comm. No. 813, J. Opt. Soc. Am. **31**, 651–678 (Nov. 1941).
(65) "Control of photographic printing by measured characteristics of the negative" (with C. N. Nelson), Kodak Research Lab. Comm. No. 874, J. Opt. Soc. Am. **32**, 558–619 (Oct. 1942).

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Psychophysics and Photography*

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THE term "psychophysics" is familiar to many members of this audience and the subject matter and methods of this science are well known to them. However, there may be some present this evening whose contact with this field is more remote and who are unaware of the broad possibilities for the application of psychophysical methods to problems not customarily treated by other than conventional methods of the physical sciences. It is desirable, therefore, to begin with a general discussion of the field embraced by the psychophysical science and by a brief history of its development since its birth over a century ago. It is hoped that the initiated present tonight will bear with the speaker while he reviews briefly the fundamental ideas of the science.

By its structure, the word psychophysics denotes a science dealing with problems lying between or common to the fields of psychology and physics. It might be supposed therefore that the subject matter and methods involved should be equally well known to the workers in these two broad fields of scientific endeavor. However, it must be confessed that psychophysics has been nurtured largely in the fold of psychology and that psychologists have been much more concerned with the application of psychophysical methods than have physicists who, perhaps to their discredit and disadvantage, have surrendered almost the entire field of psychophysics to the psychologist. This situation is indicated by the definition which Troland[1] has proposed in his discussion of the problems and methods of modern psychology: "Psychology is the systematic description of immediate experience in its own terms, and the formulation of its manner of determination by features of the physical system, or any other demonstrable factors lying outside of experience itself." It is obvious from this that Troland recognized a clear-cut division of modern psychology into two parts. The first phrase of the definition manifestly denotes that part of the science commonly referred to as *introspective psychology*, while the second phrase covers the field embraced by *psychophysics*. In harmony with Troland's statement, this latter field may be said to be concerned largely with the subjective evaluation of physical stimuli. In this discussion, the word "physical" is used in its broad sense embracing all physical sciences, such as chemistry, physics, physiology, astronomy, geology, etc.

For a clear understanding of the relative geographical positions of nations, provinces, cities, etc., a map is almost indispensable. In like manner a "map" which takes the form of a rather fanciful schematic diagram will assist materially in a comprehension of the relationship between physical, psychophysical, and psychological realms. Figure 1 is such a diagram. This is taken from the forthcoming report of the Colorimetry Committee of the Optical Society of America (a part of which has already been published[2]) and is used with the consent of its chairman who acknowledges his culpability in not having sought the consent of other members of his committee, and for this he herewith offers his apology.

The lower rectangle of Fig. 1 includes the elements that may be considered purely physical in character. Thus a "light" source *1* emits radiant energy which, after propagation through the intervening space, impinges upon an object *3*. The quality and intensity of this radiant energy may be modified in transit from *1* to *3*. This modification is indicated schematically as concentrated at *2*. Some of the radiant energy incident on the object *3* may be absorbed, some transmitted, and some reflected. The reflected radiant energy travels through space, possibly suffering modification of quality and quantity, and enters the eye through the cornea *5*. An

* Address delivered by the recipient of the Frederic Ives Medal on the occasion of its presentation at the Twenty-Eighth Annual Meeting of the Optical Society of America, Pittsburgh, Pennsylvania, on October 8, 1943.

[1] L. T. Troland, *The Principles of Psychophysiology* (Van Nostrand, New York, 1929).

[2] Colorimetry Committee Opt. Soc. Am., J. Opt. Soc. Am. **33**, 534 (1943).

image of the object is formed by the crystalline lens *6* on the retina at *8*. The retinal receptors, the rods and cones, are stimulated by the arrival of this radiant energy and as a result nerve impulses are generated which travel along the optic nerve *10* into synaptic centers, specifically the external geniculate body, located in the subcortical regions of the cerebrum. Further conduction of the nerve impulses takes place through the synapses of the geniculate body and along nerve fibers leading into the sensory projection areas *13* of the cerebral cortex. In the diagram the subcortical portion of the cerebrum is represented by the irregular area *12*, while the cortical parts of the cerebrum are designated by the areas *13*, *14*, *15*, and *16*. No effect of the incoming (afferent) nerve impulses arises in consciousness until the sensory projection area is reached. When these nerve impulses reach the sensory projection area *12*, they give rise to visual sensations. The further propagation of nerve impulses from the sensory projection areas into the association areas *14* of the cortex may give rise to a more complex psychical phenomena termed perception. Perception occurs not entirely as a result of the immediate deliverance from one or more sense organs, but is conditioned to a considerable extent by other cortical activities dependent upon previous stimulation which, for convenience, may be referred to as stored memory records.

Further propagation of nerve impulses within the associational areas *15* may produce a number of other psychical aspects of mental life, such as thought, feeling, attention, imagination, association, emotion, etc. Nerve impulses from these areas of the cortex may be propagated into the motor projection area *16*, giving rise to efferent nerve impulses which are conducted outward along the motor nerves, reaching the end-plate processes *18*, located in muscle fibers or the glands *19*, and thus produce an effect *20* on the organism or its environment. Everything that occurs from *1* to *8* is of a physical or objective

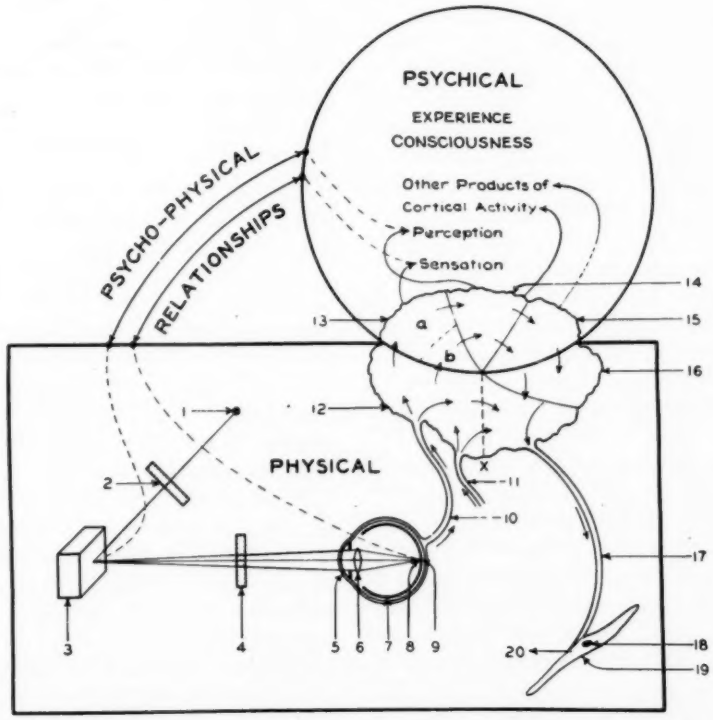

FIG. 1. Schematic diagram representing categories of visual psychophysics.

nature. The interrelations between these phenomena can be determined by conventional physical methods and expressed as physical functions. Thus the characteristics of the image *8* formed on the retina may be completely defined in purely physical terms. These characteristics of course include size, shape, and position of the retinal image, as well as the intensity and quality of the radiant energy at all points in the image. The entire chain of events leading up to the formation of, and including the image on, the retina may be designated as the *stimulus process*. The *response process* begins in or around the receptor cells (rods and cones) with the effect which occurs as a result of the absorption of radiant energy and continues inward along the afferent nerve fibers to the central nervous system and thence outward along the efferent nerve fibers terminating with a muscular or glandular reaction and its effect on the organism or its environment. This chain of physiological events *9* to *20* is called the *physical response arc*. The immediate effect in consciousness of the arrival of visual sensory nerve impulses in the sensory projection areas of the cerebral cortex is a visual sensation. This sensation must also be regarded as an aspect of the response to the stimulus, but this response is of a nature totally different from the nerve impulses themselves. This is a subjective or psychical (psychological) aspect of the response. Visual perception, which arises as a result of further propagation of nerve impulses within the cerebral cortex, is also a psychical aspect of response, although the character of the visual perception is not determined entirely by the contemporary visual sensory nerve impulses. Other aspects of the subjective response are thought, feeling, imagination, etc.

The phenomena thus far mentioned may be divided into two very general categories, namely, physical and psychical. This division is illustrated, in Fig. 1, by placing in the lower rectangle all the physical items and in the circle above all of the psychical phenomena. The fact that parts of the cerebral cortex *13*, *14*, and *15* are shown within the area enclosed by the circle, must not be interpreted to mean that the cortex itself and the nervous activity within this cortex are psychical phenomena; they are physical structures and functions, the psychical phenomena being the subjective aspects of these activities.

A correlation may be established between any pair of the items in the visual stimulus process, *1* to *8* inclusive, and it is conceivable that a relationship may be established between any one of these items and any item within the physical response group or between any two items within this group. These are purely physical relationships which, together with theories constructed to correlate or to explain them, constitute a part of physical science.

By suitable techniques, such as introspection, it is also possible to establish relationships between the various psychical components of experience. This constitutes the field of introspective psychology.

Psychophysics is concerned with the formulation of relations between the various elements of the physical processes and one or more aspects of the subjective (psychical) response. As indicated in Fig. 1, a relationship may be established between the characteristics of the visual image and the sensation arising therefrom, or between certain aspects of an object and the perceptual psychical response. These belong within the realm of psychophysics.

HISTORICAL

The exploration of the history of a science, especially its genesis, is always interesting. We shall therefore look briefly at the foundations of this science of psychophysics, but it will be quite impossible in the time at our disposal to give much attention to the details of the elaborate edifice which has been erected on these foundations.

The term *psychophysics* was coined by Gustave Theodore Fechner (1801–1887), professor of physics in Leipzig. He is generally regarded as the founder of this science which was first brought to the attention of the scientific world by the publication of his book *Elements of Psychophysics* in 1860.[3] Fechner defined psychophysics in these words: "An exact science of the functional relations or relations of dependency between body and mind." This might be a satisfactory definition of psychophysiology as we understand it today, but it is hardly broad

[3] G. T. Fechner, *Elemente der Psychophysik* (Breitkopf & Härtel, Leipzig, 1860).

enough to include the science of psychophysics as it is now conceived. As a matter of fact, Fechner's work shows that he regarded psychophysics as embracing relationships between items of the physical world, quite outside the body, and aspects of experience. Troland's definition provides a clearer concept of psychophysics in its present state of development, which deals with the formulation of the manner in which experience is determined by elements of the physical system.

Some work had been done in this field at an earlier date by E. H. Weber (1759–1878), professor of anatomy in the University of Leipzig. His experimental results, later formulated and known as Weber's law, were published in 1834. In 1846 Weber's article "Der Tastsinn und das Gemeingefühl"[4] presented definite evidence in the field of cutaneous pressure, lifted weights, and visual distances, supporting the relationship $\Delta R/R$ = a constant, where ΔR is the change in the stimulus R which produces a just noticeable change in the resulting sensation. This is Weber's law.

Pierre Bouguer,[5] a French investigator, had observed as early as 1729 that two surfaces differing in brightness by 1.6 percent were just noticeably different, even though the absolute brightnesses varied over a relatively wide range. This perhaps is the earliest psychophysical data relative to the "contrast sensitivity" of the human eye.

These earlier contributions of course were more or less fragmentary and there is little doubt that Fechner's contribution is of major importance and really constitutes the first systematic development of the science of psychophysics. Utilizing the work of Weber, supplemented by his own observations, he formulated the relationship which is now generally known as the Weber-Fechner law. He expressed this in a mathematical form, derived by the use of classical mathematical operations, as follows:

$$S = K \log R.$$

This formula was interpreted by Fechner as follows: "The magnitude of sensation S stands in relation not to the absolute magnitude of the stimulus R but to the logarithm of the magnitude of the stimulus, when the unit of stimulus is defined as its liminal value r, i.e., as that magnitude at which sensation appears and disappears. If we call the value R/r the fundamental stimulus value, we may say that S-magnitudes are proportional to the logarithms of the corresponding fundamental R values. If we make $r = 1$, we may say simply that sensation is proportional to the logarithm of the stimulus."[6] The publication of Fechner's work *Elements of Psychophysics* provoked a storm of controversy which lasted for many years and the echoes still may be heard. In particular, his mathematical treatment of psychophysical data and the interpretations drawn therefrom were questioned and denied by many authorities in the field. Quite apart from the validity of Fechner's treatment and of the conclusions drawn therefrom, he unquestionably did establish the possibility of formulating definite and meaningful relationships between objective stimuli and subjective response. In the words of Titchener[6] "We owe to Fechner the comprehensive development and theoretical interpretation of the metric methods . . . the concept of the *DL*,* and the thorough exploitation of the differential sensitivity—a number of psychophysical investigations, which are important both extrinsically, for their results, but also and more especially intrinsically, as undertaken with an overt psychophysical purpose and carried out by means of the psychophysical metric methods. . . . We owe to Fechner the sole conception of mental measurement that has stood the test of 'inside' criticism: the idea of measuring S by laying-off units of its own kind, and of standardizing the units by aid of their correlated R. . . . The thesis that equally noticeable (including 'just' noticeable) differences of S are equal, i.e., may be employed as S-units, is, as we shall presently see, accepted by the majority of experimental psychologists at the present time. We do not, of course,—the point has been made several times in what

[4] E. H. Weber, "Der Tastsinn und das Gemeingefühl," Wagner's *Handwörterbuch der Physiologic*, Vol. 3, Part 2, 481 (F. Vieweg und Sohn, Brunswick, 1846). De Tactu: annotationes et physiologicae. Monograph (1834).

[5] M. Bouguer, *Essai d'Optique sur la gradation de la lumière* (Jombert, Paris, 1729).

[6] E. B. Titchener, *Experimental Psychology* (Macmillan, New York, 1905), Vol. 2.

* Difference limen.

precedes,—regard the single *S* as a sum of *S*-pieces, or the *S*-unit as a little bit of a larger *S*-magnitude. Fechner did: though Fechner, too, had floating ideas of 'distance' measurements in psychophysics. But our modern view is simply a reinterpretation of Fechner's Massprincip, not an entirely new view, put in place of Fechner's."

The three metric methods elaborated by Fechner are as follows: (a) the method of just noticeable differences, (b) the method of average errors, and (c) the method of right and wrong cases. He did not actually originate these methods because (a) had been used previously by Bouguer (1729),[5] by Delezenne (1826),[7] and by Weber (1831);[4] (b) had been employed by Stinheil (1837),[8] and (c) was used by Vierordt (1852).[9] However, Fechner undoubtedly made a very outstanding and valuable contribution in revising and coordinating these methods and of bringing them to the attention of a large audience.

Some of the criticisms levelled at Fechner's methods and interpretations were met by J. L. R. Delboeuf (1831–1896).[10] He introduced the concept "sense distance" or "degree of sensible contrast" and maintained that although sensations cannot be regarded as complex magnitudes they may, in fact, be conceived as separated by variable distances which can be compared as "greater," "equal," or "less." These three kinds of judgments provide the necessary material for the application of psychophysical methods.

Georg Elias Müller (1850–) prepared an exhaustive criticism of Fechner's work which was published in 1878 under the title *Zur Grundlegung der Psychophysik*.[11] This was a constructive contribution and had a profound effect upon the attitude of workers in this field toward the psychophysical point of view. In fact, it induced Fechner to write a revision of his *Elements* under the title *Der Hauptpunkte der Psychophysik* which was published in 1882.[12]

Johannes Müller (1801–1858),[13] whose work was essentially of a physiological character, made worthwhile contributions to the science of psychophysics, his field lying on the border line between physiology and psychology. Among the pupils of Müller were two eminent psychophysicists, Emil Du Bois-Raymond (1818–1886)[14] and Hermann von Helmholtz (1821–1894). The latter achieved fame as one of the greatest scientists of the 19th century. His treatises on physiological optics[15] and physiological acoustics[16] stand as classics which have probably never been surpassed in point of comprehensiveness and excellence.

Many important contributions were made to the science of psychophysics by several followers of Helmholtz, among which may be mentioned Johannes von Kreis (1853–1928),[17] Arthur König (1856–1901),[18] and E. Brodhun (1860–).[19]

As stated previously, psychophysics has been developed largely in laboratories devoted to experimental psychology and the record of the growth of this science cannot be made complete without a mention of several men usually associated with psychology, but whose work actually involved the application and elaboration of psychophysical methods to the problems of that science.

Wilhelm Wundt (1832–1920) was trained in the medical sciences and became assistant in physiology to Helmholtz in 1857 while Helmholtz was actively engaged in the experimental work, reports of which were later incorporated in his *Physiological Optics*.[15] In the succeeding years Wundt made good use of his knowledge of physiology and psychophysics and applied it to the problems of experience in which he became deeply interested. He established the first labo-

[7] C. E. Delezenne, Rec. trav. soc. sci. de Lille (1826–27).
[8] K. A. Stinheil, "Elemente der Helligkeitsmessungen am Sternenhimell," Abh. d. k. bayr. Akad. d. Wiss. math. phys. Kl. II (1837).
[9] K. Vierordt, Arch. f. physiol. heilkunde 9, 844 (1852).
[10] J. L. R. Delboeuf, *Examen critique de la loi psychophysique* (Germer Baillière et Cie, Paris, 1883); Rev. phil. V, 192 (1878).
[11] Georg E. Müller, *Zur Grundlegung der Psychophysik* (Th. Grieben, Berlin, 1878).
[12] G. T. Fechner, *Der Hauptpunkte der Psychophysik* (1882).
[13] J. Müller, *Zur vergleichenden Physiologie des Gesichtssinnes des Menschen und der Thiere* (Leipzig, 1826).
[14] E. DuBois-Raymond, *Untersuchungen über thierische Elektrizität* (Berlin, 1849).
[15] H. von Helmholtz, *Physiological Optics*, Am. Ed. (Opt. Soc. Am., 1924).
[16] H. von Helmholtz, *On the Sensations of Tone*, Trans. by A. T. Ellis, English edition (London 1885); fourth edition (Longmans, Green & Company, New York, 1912).
[17] J. von Kries, *Abhandlungen zur Physiologie der Gesichtsempfindungen* (Hamburg and Leipzig, 1897).
[18] A. König, *Gesammelte Abhandlungen zur physiolog. Optik* (Leipzig, 1903).
[19] E. Brodhun, "Experimentelle Untersuchungen über die psychophysiche Fundamentalformel in Bezug auf den Gesichtssinn," Sitz. d. preuss. Akad. Wiss. Berlin, 917 (1888).

ratory of experimental psychology in the world in Leipzig in 1879, and while Wundt's interest was much broader than the field of psychophysics, the development of that science was advanced materially through the work which he actually did. His outstanding contribution to the literature of the subject was *Grundzüge der physiologischen Psychologie* in 1872.[20]

The evolution of psychology in England followed somewhat different lines than in Germany and it was not until the latter part of the 19th century that much attention was given to experimental psychology and the necessarily closely associated psychophysics. Among the English authors who approached their science from the experimental point of view may be mentioned G. F. Stout,[21] whose publications appeared during the last decade of the 19th century and the first of the 20th, and James Sully, a contemporary of Stout, who exhibited a very marked tendency toward experimental methods and the viewpoint of the natural sciences. Several publications from his pen appeared between 1874 and 1892. One of these, *Sensation and Intuition*,[22] contains an exposition of psychophysics.

In America little attention was paid to psychophysics before the latter part of the 19th century. James McKeen Cattell was among the first group of workers in Wundt's laboratories. His reports published in *Mind* (London) in 1881 were the first covering the work done in Wundt's laboratory to reach English and American readers. The first laboratory for experimental psychology in America was established by Stanley Hall (1846–1924) in 1883, who was at the time professor of psychology and pedagogy at the Johns Hopkins University. The first chair exclusively of psychology was established in 1888 at the University of Pennsylvania. This was occupied by Cattell and at that time a laboratory for experimental work in this field was founded. America soon thereafter took the lead in experimental psychology. The rapid progress in this field was due largely to William James (1842–1910). He was

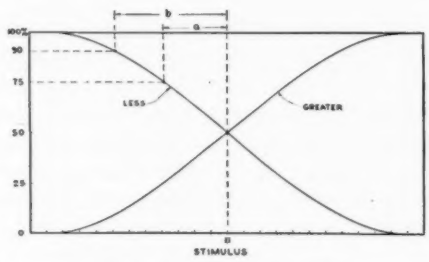

FIG. 2. Psychometric function.

trained in medicine, physiology, and anatomy, and became professor of philosophy at Harvard in 1899. He was a man of very broad interests. In 1890 he published *Principles of Psychology*,[23] treating comprehensively the physiological, psychophysical, and analytical aspects of psychology. Associated with James at Harvard was Hugo Munsterberg (1863–1916) who was an able experimentalist and applied psychophysical methods to many of the problems to which he made outstanding contributions.[24]

Perhaps one of the greatest experimental psychologists who fostered the development of psychophysical methods was E. B. Titchener (1867–1927).[25] For many years he was director of the psychological laboratory of Cornell University, from which have come many valuable contributions to the science of psychophysics.

Running through all of these developments subsequent to Fechner's pioneer work, we find the three metric methods which he proposed and discussed. All of these as a matter of course have been elaborated and refined. This is particularly true of the method of right and wrong judgments. Quite recently it has been elaborated by F. M. Urban[26] to the form now very familiar to everyone in this field and known as the psychometric function.

A psychometric function is illustrated in Fig. 2. There are many ways, differing in detail, of applying the psychometric functions to psychophysical problems. The significance, however, can be described rather simply, as follows: Along

[20] W. Wundt, *Grundzüge der physiologischen Psychologie* (1872), translated by E. B. Titchener (London, 1904).
[21] G. F. Stout, *Manual of Psychology* (Hinds & Noble, New York, 1899, 1901).
[22] J. Sully, *Sensation and Intuition* (K. Paul, London, 1880).
[23] Wm. James, *Principles of Psychology* (Holt, New York, 1890).
[24] Hugo Munsterberg, *Psychology, General and Applied* (Appleton, New York, 1914).
[25] E. G. Boring, Am. J. Psychol. **38**, 489 (1927).
[26] F. M. Urban, *Applications of Statistical Methods to Problems of Psychophysics* (Psychological Clinic Press, Philadelphia, 1908).

the bottom of the figure is established a scale of stimulus values and the point *O* represents a standard or reference stimulus. The observer is presented with the standard stimulus and a variable stimulus which differs from it by some known amount. He is then asked to state whether the variable stimulus produces a sensation which is greater or less than that produced by the standard stimulus. The variable stimulus is then changed using a series of values differing by various amounts from the standard stimulus. A large number of observations are made and for each value of the variable stimulus the percentage of right and wrong answers is plotted. Through the points thus obtained curves designated as "less" and "greater" are drawn. When the variable stimulus is equal to the standard stimulus, the number of judgments "less" must be equal to the number of judgments "greater." Hence the two psychometric functions must pass through the 50 percent point. When the variable stimulus is sufficiently greater than the standard stimulus, the judgment "greater" will be obtained in 100 percent of the cases; while when it is sufficiently less than the standard stimulus 100 percent of the judgments will be "less." It is reasonable to seek some relation between the psychometric function and the just noticeable difference. For instance, in such cases as illustrated by Fig. 2, the 75 percent points on the psychometric function are frequently taken as determining the just noticeable difference. However, under some conditions it may be more desirable to use some other value. Thus if one is interested in knowing the stimuli difference which is noticeable with certainty, we might choose to use the stimulus increment corresponding to the 100 percent value. Since the curve approaches the 100 percent line asymptotically, this stimulus value cannot be determined satisfactorily and in some cases the 90 percent level may be preferable.

Noteworthy contributions of relatively recent date to this subject are *Psychophysik* by W. Wirth (Leipzig, 1912) and *Essentials of Mental Measurements* by William Brown and G. H. Thompson (Cambridge, 1921).

I am reluctant to close this historical review without mentioning a man whose contribution to the science of psychophysics perhaps has not been adequately recognized. I refer to Dr. P. G. Nutting (1873–), a physicist who, from 1903 to 1912, was chief of one of the optics sections at the Bureau of Standards and was chief physicist of the Kodak Research Laboratories from 1912 to 1916. In addition to other optical problems he was intensely interested in vision and applied psychophysical methods to the determination of various visual functions, such as the "visibility" of radiant energy; visual sensitivity to differences in brightness, wave-length, and purity; and visual glare. In the period between 1903 and 1916 he published many contributions to this field.

To Dr. Nutting I owe a debt of gratitude for his guidance and inspiration in the early days of my scientific career. It was a rare privilege to work under his direction. In these Laboratories he initiated work on several psychophysical problems related to vision and supervised and trained a group of workers in this field. Julian Blanchard and Prentice Reeves were among these and their publications have contributed to our knowledge of visual functioning. It is doubtless due to Dr. Nutting's example and guidance that I became interested in the application of psychophysical methods to photographic and visual problems. In fact, it may very well be that without the encouragement and inspiration which I drew from Dr. Nutting I would not be standing before you this evening as the recipient of the Frederic Ives Medal.

IMPORTANT PSYCHOPHYSICAL CONCEPTS AND RELATIONSHIPS IN VISION

To lend clarity to our concept of the psychophysical relationship, a few familiar examples will be mentioned. Perhaps one of the best known and most important of such relationships is that known as the luminosity function. This is shown in Fig. 3. The ordinates of this curve are proportional to the reciprocals of the stimulus intensities, or radiant energies of different wavelengths, required to produce equal sensation intensities. This relationship is sometimes said to represent the spectral sensitivity of the human visual mechanism and it is sometimes referred to as the luminous efficiency of radiant energy. The latter statement implies that it represents a characteristic of radiant energy, and the former

that it is an attribute of the responding organism. As a matter of fact, it is neither, but something of a medial nature partaking of the characteristics of both the responding organism and the physical stimulus. It is a purely psychophysical concept.

Illuminating engineers have chosen to define light as a psychophysical concept in these words: "For the purposes of illuminating engineering, *light* is radiant energy evaluated according to its capacity to produce visual sensation." This definition has been approved by the American Standards Association[27] and has further been affirmed in substance by the colorimetry committee of the Optical Society of America. The significance of this definition is cogently stated by Troland:[1] "It is evident that light, as thus conceived, is neither a physical nor a psychical quantity. It is, rather, a psychophysical notion, involving c.g.s. units, on the one hand, and a psychical dimension . . . on the other. Light can neither be identified with brilliance* nor with radiant energy. It has the properties of both, taken together. Insofar as physicists and engineers think in terms of this notion, they are basing their thought in a very definite way upon psychological principles." Physicists and engineers frequently forget or wilfully ignore the psychophysical nature of this relationship, which is one of the corner stones of the practice of illuminating engineering. Some authors who ignore the human significance of these data, in order to create an illusory simplicity for the purposes of engineering, look upon the concepts defined by these standardized psychophysical relationships as purely physical because the fact of standardization has been substituted for the criterion of subjective validity. Some authors regard brightness, for instance, as a derived physical quantity since it can be computed from the spectroradiometric data of the emitting surface by use of the standard psychophysical data (photopic luminosity curve) without the necessity for any visual observation or measurement. Troland refers to this attitude of mind in the following words:[1] "There is a type of intellect which is frequently encountered among physicists that appears to be afflicted with a sort of *experience aphasia*, causing its thought completely to gloss over the immediate data of experience, so as to pass at once to the inferential conceptions of physical science. Such an intellect, although constantly employing the experiential data as a basis for thought, never explicitly recognized their reality."

Not only has light been assigned to the psychophysical category of concepts, by almost universal consent, but *color* also has been placed in this same category by the OSA colorimetry committee which has formulated the following definition of color: "Color consists of the characteristics of light, other than spatial and temporal inhomogeneities; light being that aspect of radiant energy of which a human observer is aware through the visual sensations which arise from the stimulation of the retina of the eye." By the terms of this definition, color is not identified with radiant energy, nor is it identified with sensation. Standardized psychophysical relationships (Fig. 4) based on color mixture data have received international approval and these, together with spectroradiometric data relative to a sample of radiant energy, permit the measurement of color in psychophysical terms. All of colorimetry (the measurement of color) therefore, is based on psychophysical relationships. It may be of interest to note in passing that the idea of regarding color as belonging neither to the physical nor psychical categories of concepts was conceived many years

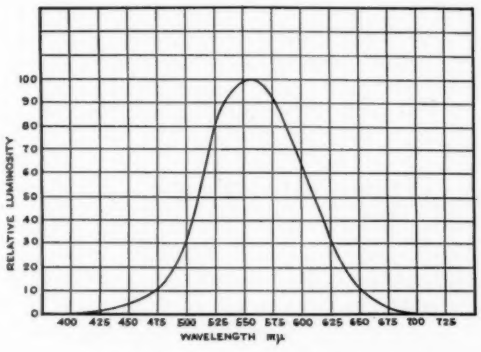

FIG. 3. Photopic luminosity curve.

[27] ASA Z7.1–1942, American Standards Association, New York.

* By brilliance he means an attribute of visual sensation.

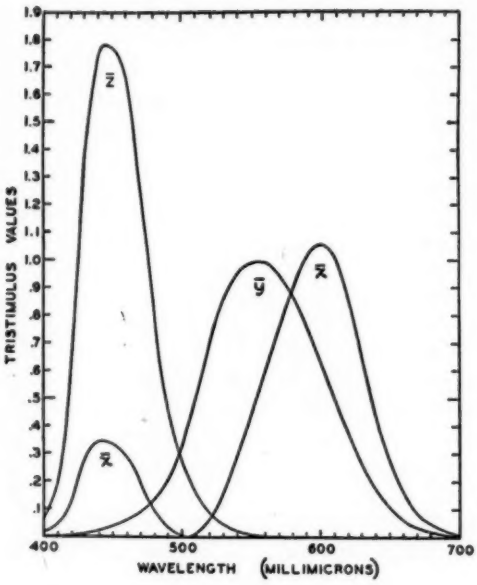

FIG. 4. Psychophysical functions derived from color-mixture data.

ago, as indicated by the following notes from Field:[28] "According to common apprehension, colours are inherent or substantial qualities of bodies, or material things. The more speculative have ascribed them to the sense of vision only, regarding them as intellectual affections through the organs of sight; while a third class of inquirers has assigned them variously a medial stage between vision and colored objects, *viz.*, in light."

The adoption of the psychophysical position not only admits but asserts that color has an inherently medial nature implying at once an appraiser, the human observer, and an object of appraisal, radiant energy.

A final example of well-known psychophysical relationships is that which shows the sensitivity of the visual mechanism to brightness differences. While the two already mentioned are of considerable importance in the application of psychophysical methods to some photographic problems, the contrast sensitivity function is of immediate and vast importance to many photographic problems. As indicative of this relationship, Fig. 5* is shown. This represents a summary of a considerable mass of data as made by Moon.[29] It is evident that sensitivity to brightness differences depends on the conditions of observation. Thus the data attributed to König, Blanchard, etc., indicate that the sensitivity increases up to a maximum, remains practically constant for a rather great range of brightness values, and then begins to decrease. These observations apparently were all made by using a two-part visual field subtending a relatively small visual angle (possibly two or three degrees and therefore just about covering the fovea), the remainder of the visual field being of very low brightness. The data of Stiles[30] represent a very different relationship between contrast sensitivity and brightness level. These measurements were made with the visual field surrounding the test fields adjusted at all times to the same brightness as the test fields themselves. The sensitivity increases steadily and more rapidly with no region of constant sensitivity and apparently no tendency to decrease at the highest brightness levels used by these investigators.

Much work has been done on this sensitivity function, but even so it is difficult to apply the available data to many of the photographic problems with the assurance that the results will be valid. This follows because it is difficult to be certain that the published data represent observational conditions sufficiently similar to those existing in the comparison of photographic print quality with the brightness conditions which existed when the original of the reproduction was viewed.

The necessity of carefully specifying observational conditions when making psychophysical measurements is not peculiar to this field of endeavor, but exists also in all the physical sciences. In this respect the difference between psychophysical and physical measurements is one of degree and not of kind.

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[28] G. Field, *Chromatography or a Treatise on Colours and Pigments* (Chas. Tilt, London, 1835).

* Reproduced from *The Scientific Basis of Illuminating Engineering*, P. Moon, by permission of McGraw-Hill Book Company, New York.

[29] P. Moon, *Scientific Basis of Illuminating Engineering* (McGraw-Hill Book Company, New York, 1936).

[30] W. S. Stiles, Proc. Roy. Soc. **104B**, 322 (1929).

THE PSYCHOPHYSICAL EVALUATION OF PICTURE QUALITY

The word "quality" is almost invariably encountered in any discussion of the merits of a picture made by photographic processes. For instance, such statements as "this picture exhibits excellent quality" or "the quality is very poor" are frequently heard. Many difficulties arise when an attempt is made to attach a single definition to this word. As a matter of fact, it is used in different senses by different individuals and even by the same individual in referring to various aspects of the appearance of a picture. One aspect of this quality concept depends upon the way in which the scene is lighted. If the scene be an exterior one illuminated by natural sources, this aspect of quality is determined by the skill of the photographer in choosing the time of day and point of view. Another aspect of picture quality depends largely upon compositional factors, such as the position of objects relative to each other within the scene, the balance between light and shade, etc. It is evident that the factors which control these aspects of quality are not functions of the photographic processes *per se*. They may be referred to as the aesthetic or compositional qualities.

The expression "photographic quality" is frequently used in referring to the degree of perfection with which the photographic picture reproduces in the mind of the observer the subjective impression which he received when looking at the original. Obviously this statement cannot apply if the observer has never seen the original, but doubtless accumulated experience with similar objects has built up in his mind a very definite impression of how he thinks the object would have looked had he seen it.

While it seems reasonable to assume that photographic quality is dependent largely upon the *fidelity* with which the subjective evaluation of brightness and brightness differences are reproduced, it is not safe to assume that a precise reproduction of the original subjective impression is that which the observer would accept as best possible photographic quality for he might be more pleased with a reproduced subjective impression into which some distortion has been introduced.

Photographic quality is dependent upon two factors: the exactness (or inexactness) with which the brightness and brightness differences in the original are reproduced in the illuminated positive as viewed by the observer; and certain characteristics of the observer's visual sensory and perceptual mechanisms. It is with these features of photographic quality, rather than with the aesthetic characteristics of the picture, that this discussion will deal.

In the evaluation of the photographic aspect of picture quality, then, it may be desirable to consider the two parts separately, thus necessitating (a) a comparison of brightness and brightness differences in the object with those in the illuminated positive, and (b) an evaluation of the subjective impression resulting from an inspection of the illuminated print in terms of that obtained when viewing the original. In any case, the problem includes the consideration of *subjective* factors in addition to those of a strictly *objective* or *physical* nature.

While our knowledge of the psychophysical relationships involved in this problem is sufficient to permit the prediction of certain characteristics of the subjective response resulting from a specified stimulus, it is not sufficient to provide a complete solution. Purely introspectional methods might be resorted to, but unfortunately such methods do not yield results specifiable in

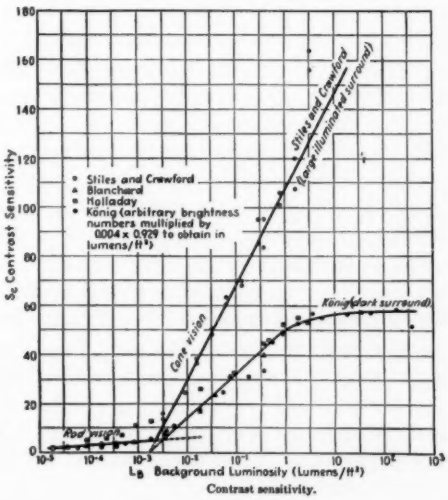

FIG. 5. Contrast sensitivity.

quantitative terms. It seems more expedient, therefore, to determine the relationships between the brightness and brightness differences existing in the original and those of the illuminated positive and then to express photographic *quality* (or selected aspects thereof), as determined statistically from the direct judgments of a large group of observers, in terms of the measured physical characteristics.

From the physical point of view, a *perfect* reproduction, that is perfect *objective* photographic quality, is obtained when the brightness and brightness differences in the illuminated positive are identical with those in the object or scene photographed. However, such a positive would, in general, fail to reproduce with satisfactory fidelity the subjective impression of the original because of the difference (usually very great) between the visual field environment of the original and that of the reproduction. The portion of the visual field photographed usually includes only a small part of the total visual field. When the reproduction of this small portion is viewed in entirely different surroundings (usually at a very much reduced brightness level) the brightness and brightness differences required to reproduce the original subjective impression may differ greatly from the brightness and brightness differences existing in the original.

The theoretical relationships and practical methods for expressing in physical terms how closely the brightness and brightness differences in the original can be reproduced by the photographic processes are well known. When these methods are applied in practice, it is usually found to be impossible, with the available photographic materials, to obtain an exact physical reproduction of many objects. This is particularly true when the positive is made on a reflecting surface, such as paper, which must be viewed by reflected light. Compromises, therefore, must be made. The magnitude and character of the compromises which will produce the least detrimental effect upon the final reproduction quality cannot be determined by purely physical methods but must necessarily be established in terms of their subjective effects. The problem therefore lies squarely in the realm of psychophysics.

It is obvious that the physical characteristics of the photographic reproduction are determined not only by the characteristics of the negative materials, the amount of exposure given in making the negative, and the way in which the exposed material is developed, but also by the characteristics of the positive material, its printing exposure, and its treatment in development.

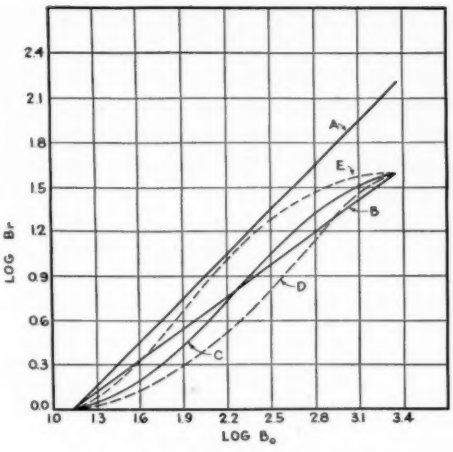

FIG. 6. Tone reproduction curves.

Let us now consider a typical case. The average brightness scale BS_0 (ratio of maximum to minimum object brightness) of exterior scenes has been found to be 160, log $BS=2.20$.[31] A typical tone reproduction curve obtained for such a scene by using good photographic techniques (correct exposure, development, printing, etc.) is shown as the solid curve C in Fig. 6. The positive material used was a developing-out paper having a semi-matte surface of a "contrast" grade to "fit" the negative. The useful density scale of this material is 1.60, which is considerably less than the log BS_0 of the scene. This curve C shows the relation between print density D_p and log object brightness log B_0. It is apparent that with this material the brightness scale of the print is considerably less than the brightness scale of the original. Hence, there must be, on the average, a compression of the brightness differences existing in the scene. Perfect objective tone reproduction is represented by the straight line A drawn at an angle

[31] L. A. Jones and H. R. Condit, J. Opt. Soc. Am. **31**, 651 (1941).

of 45 degrees to the log B_0 axis. The gradient at any point on curve C is given by $dD_p/d \log B_0$. The *average* gradient of the reproduction curve C in this case is not unity but 0.73. Since compression of object detail must be accepted, it might be reasonable to assume that the best possible solution would be to compress all of the object detail to the same extent. Such a reproduction is represented by the straight line B. The curve C, which represents a print of very good photographic quality, does not follow this straight line. Compression of detail is greatest in the highlight and shadow regions, while through a part of the middle tone region enhancement of brightness differences is found instead of compression.

In considering the objective aspects of this problem it seems reasonable to assume that the best method of expressing quality loss lies in the evaluation of gradient errors and, further, that the least loss of quality is associated with the least possible average gradient error. If evaluated in subjective terms, however, there is no *a priori* reason to conclude that the minimum average gradient error will result in a positive of maximum tone reproduction quality.

By modifications of the photographic technique this same scene may be rendered as shown by the dotted curve D, in which the highlight detail is more faithfully reproduced than in the case of C. This has been accomplished by a further sacrifice of shadow detail rendering. Another modification of the photographic technique may result in the curve E in which preference has been given to better rendering of the shadow detail at the expense of highlight detail. There is an almost infinite number of modifications which will yield results differing appreciably from any of those shown in Fig. 6. Thus, it is quite possible to obtain more faithful proportional rendering of object detail through the middle tone areas by further sacrifice of density scale, thus entailing further departure from the ideal average gradient of 1.0. Gradient $dD/d \log E$ is plotted as a function of log B_0 in Fig. 7. The five curves shown correspond to those in Fig. 6. Expressed in this manner perfect objective tone reproduction is represented by the curve A, for which the ordinate value is constant at 1.00. Perfect *proportional* objective tone reproduction is given by curve B for which the ordinate value is constant at 0.73. Curve C represents the gradient relationship for a print having good photographic quality. Curves D and E represent possible variations in the *distribution* of the gradient errors along the tonal scale. There seems to be no possible way of concluding *a priori* what particular distribution of gradient errors will result in the best photographic quality. We know that prints represented by the curves D and E have a lower photographic quality than C. This knowledge, however, was not obtained by objective methods but by *inspecting* prints corresponding to these conditions, a purely psychophysical procedure.

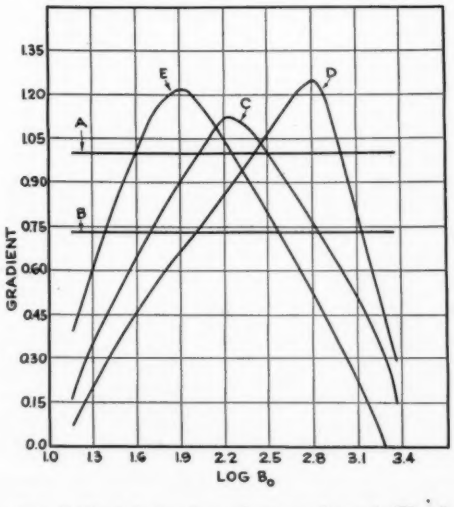

FIG. 7. First derivatives of curves shown in Fig. 6.

We shall now discuss a psychophysical method whereby the dependence of photographic print quality upon one of the many factors involved in making the picture may be determined.[32] The variable which has been chosen is the exposure given to the negative material in the camera. A series of negatives was made, beginning with a camera exposure so low as to result in a badly underexposed negative. The camera exposure was then increased by fixed steps covering a wide range of camera exposure. All of these negatives were given identical development and from each negative the best possible print was

[32] L. A. Jones, J. Frank. Inst. **227**, 297, 497 (1939).

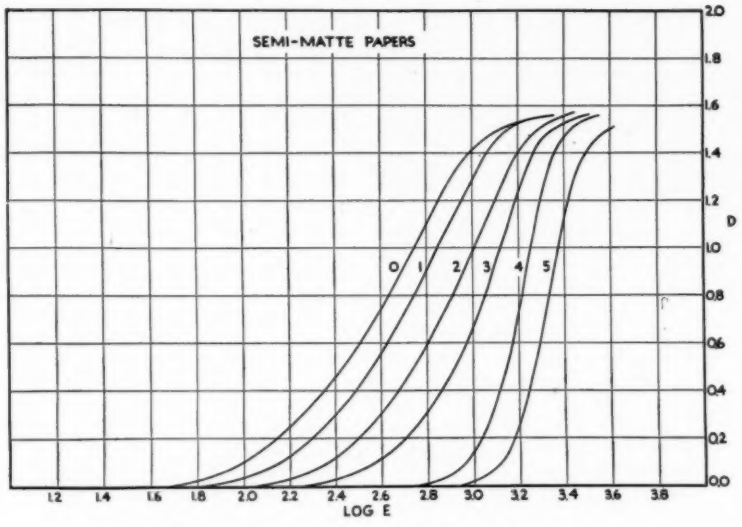

FIG. 8. Characteristic curves of developing-out papers.

made. As you are aware, it is common practice, particularly in the amateur field, when making prints from a negative, to choose a positive material best adapted to the negative in question. For this purpose manufacturers of photographic materials provide several "contrast" grades. In this work a positive material of semi-matte surface and available in six grades, as indicated by the curves in Fig. 8, was chosen.

It is not possible to predict precisely from objective data which of these grades will result in best possible print quality, and the procedure illustrated in Fig. 9 was used to obtain the best possible print from each negative. A series of prints was made on a grade which it was estimated would yield prints somewhat too "soft." The printing exposure was increased by equal increments and all prints were given the same processing treatment. The same negative was then printed on a second member of the positive material family chosen to yield prints of somewhat greater contrast, and finally on a third and possibly on a fourth grade. The processing of all prints was rigidly controlled.

These prints were then assembled on a judging easel, as shown in Fig. 9, and a group of observers were instructed to make two judgments, first the best print on each of the grades, and finally the grade giving the best quality. It should be mentioned that in making these judgments only one observer was permitted to inspect the prints at one time and each observer was given exactly the same instructions as to his duties and was never influenced in any way by comments or discussion with the individual administering the test. In this way one print for each of the series of negatives was finally obtained. Such a series is illustrated in Fig. 10. The print from the negative having received the least exposure is shown in the upper left-hand corner and camera exposure increases by equal geometrical increments progressively as indicated by the print numbers. A casual inspection, such as can be made by looking at this figure, shows that as negative exposure increases, the print quality increases, reaches a maximum and, within the exposure range shown, remains at an approximately constant level of print quality. Some judgment, perhaps not very precise, can be made of the rate at which this quality increases, and the curve in Fig. 11 shows approximately the relation between negative exposure and print quality. As drawn, it appears that quality increases rather slowly at first, then quite rapidly to a maximum beyond which there may be small fluctuations above and below a horizontal line. It seems probable that this small fluctuation arises from the discontinuous nature of the

"contrast" variation within the group of positive materials available. If this contrast were variable by very small steps, there seems little doubt that such "waves" in the quality curve would disappear.

Now having this series of prints, a more quantitative evaluation of quality can be established. The precise method of proceeding from this point depends a little upon the object of the investigation. One question which may be asked is, what is the least negative exposure which will yield a positive of some verbally describable quality level, such, for instance, as "excellent" or "just acceptable." Four series of prints obtained from four different negative materials have been judged by a group of 300 observers in an effort to establish the negative exposure corresponding to these two quality levels. The technique followed in this case was to arrange the series of prints, beginning with the one from the negative of least exposure at the left, in the order of increasing negative exposure and to instruct the observer to proceed from left to right and designate the first print which in his opinion was "just acceptable" and then to continue further along the exposure increment scale until he found the first print which he considered to be of excellent quality. The results of this judgment work, shown in Fig. 12, are plotted as frequency distribution curves. It will be noted that these frequency curves are sharply peaked and establish with considerable certainty the negative exposure which results in prints of these two quality levels.

This method of judgment does not, however, result in the establishment of the print quality-negative exposure function and for this a somewhat different procedure has been followed. The

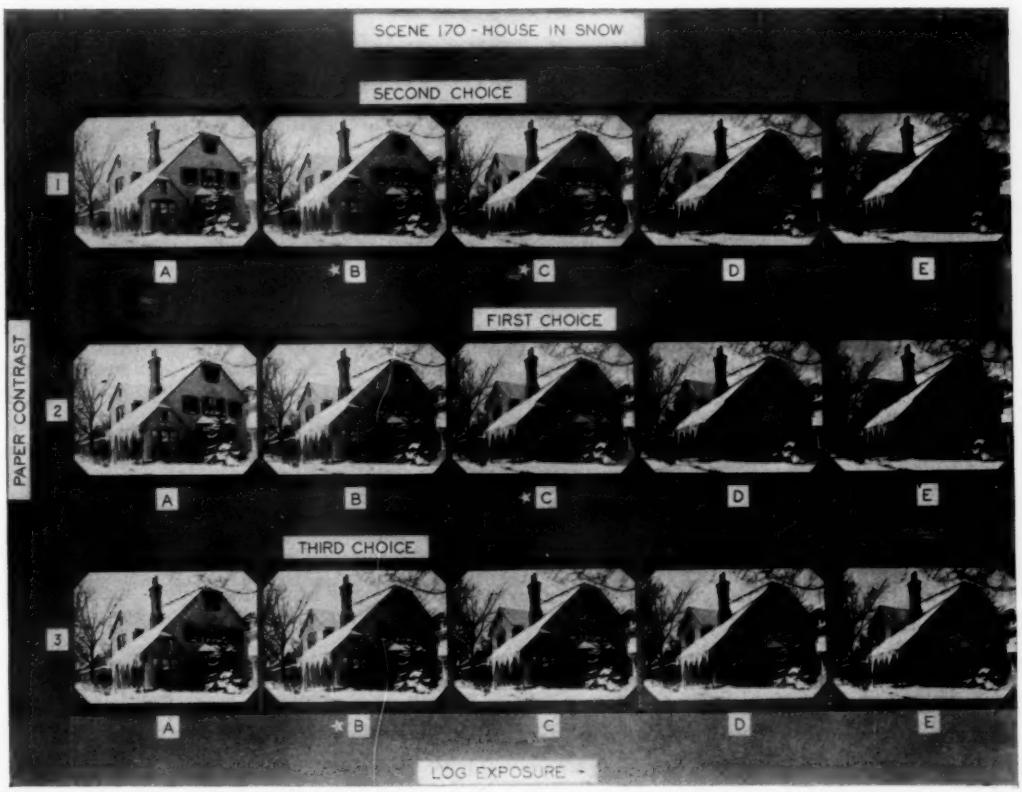

FIG. 9. Disposition of prints for judgment.

FIG. 10. Best print from each of twelve negatives.

observer is given the 12 prints, for one of the negative materials being used, in a "shuffled" condition. He is then directed to arrange these on the judging easel in a horizontal series placing the print of the poorest quality at the extreme left and the remainder in order of increasing quality to the right. When he has done this once, a record of the position of the various prints in the series is made. This is continued for a large number of observers. After a lapse of time sufficient to erase possible memories of his first judgments, each observer is asked to repeat the process. This is continued until at least a part of the observers have made the judgments several times. Some of the results are shown in detail in Table I. It will be seen that observer No. 1 made no errors in arranging the prints in the same order as the negative exposure until No. 7 had been passed. Beyond that point he was unable on subsequent occasions to repeat his arrangements of prints 8 to 12, inclusive. In the case of a large number of judgments by the same observer, this condition persisted and his arrangements of prints from 8 to 12 began to resemble a purely random distribution. Some other observers, it will be seen, agreed perfectly with observer No. 1 up to print No. 8, while the second observer began to disagree with observer No. 1 at print No. 7. However, he also was infallible in his arrangement of prints from 1 to 6, inclusive.

From an extensive group of data similar to that in Table I a numerical rating of the print quality is obtained using the method illustrated in Table II. The number of times that a print appears in each possible position is determined

and multiplied by the index number of that position. The sum of these products is then obtained and it will be seen that for prints Nos. 12 and 11 the total scores are 506 and 487, respectively.

The final summary of this method is shown in Table III. If the numbers in the first line are considered, it will be seen that they continue to increase rapidly until print No. 8 is reached, after which the totals differ but little from each other. This is interpreted to mean that for negative exposure greater than that given to the negative from which print No. 8 was made, very little, if any, increase in print quality is achieved. The fact that an observer invariably arranged the first few prints in correct order is interpreted to mean that the difference between print quality is sufficiently great so that he can always judge them in exactly the same manner. In other words, the quality difference is well above the just noticeable difference. For prints in the group 8 to 12, this ceases to be true. While there must be differences in print quality, this difference is less than the just noticeable difference and if it were not he would always arrange these prints in the same order. To obtain the final quality curve, it is assumed that the print quality of the group 8 to 12 is practically constant and the average quality for the group is arbitrarily made equal to 100 percent. On this basis the adjusted values shown in the bottom line are obtained. These values, when plotted as a function of negative exposure, give the solid curve shown in Fig. 13.

This method of judgment indicates that the quality increases from a low value according to a linear relation within the group 1 to 6, inclusive. This conclusion is not valid because the method gives no information in this region except as to the *order* and, by its very nature, cannot give any indication of the quality *interval.*

An attempt has been made to use a somewhat different judgment technique in order to remedy this situation. The observer is requested to lay out, let us say, prints 1 and 2, with some fixed distance, for instance 1 inch, between them and then to continue the process allowing the space between adjacent prints to represent his estimate of *quality difference.* This type of judgment is rather difficult to make, but surprisingly enough a moderately good agreement between observers can be obtained and a single observer can repeat

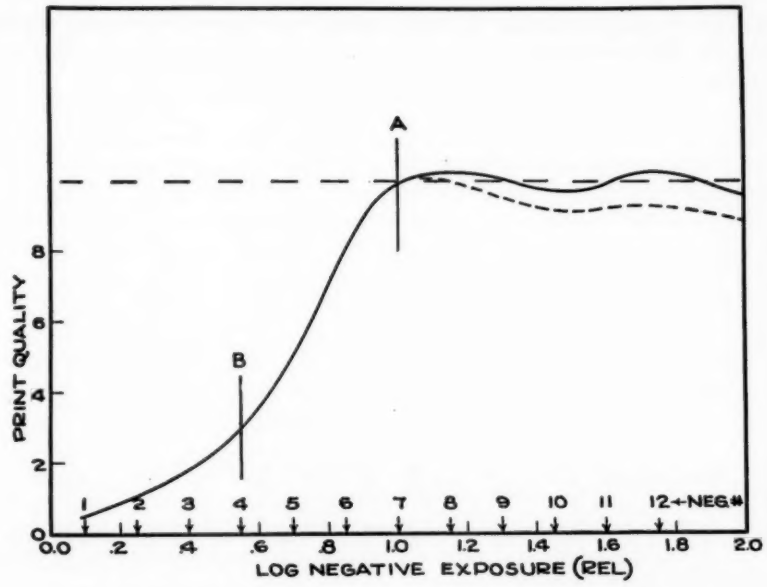

FIG. 11. Curve showing approximate relationship between print quality and negative exposure.

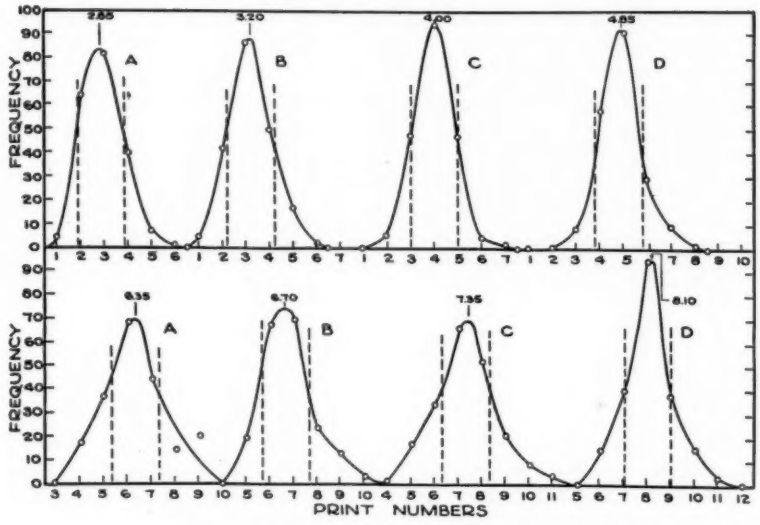

FIG. 12. Frequency distribution of print choices.

his judgments of this type with reasonable certainty. On the basis of a rather insufficient volume of judgments made in this way it is believed that the quality increases rather slowly in this region of severe negative underexposure and then more rapidly, as indicated by the dotted curve in Fig. 13.

It is obvious that the judgment data thus far obtained supply no information on the relation between the best possible photographic quality obtainable with one negative material and that obtainable when using other negative materials. If desired, however, this information can be secured by statistical judgment data obtained in a manner very similar to that already described but involving a comparison of prints in which the variable is *negative material* rather than *camera exposure*. Curves such as those in Fig. 13 should therefore be looked upon as an expression of the relation between *relative* photographic quality and camera exposure. It is possible now to attempt the evaluation of photographic quality in objective terms. An empirical relationship has been formulated[33] expressing quality in

TABLE I. Typical data. Judgment of relative print quality.

Observer	Position in series 1	2	3	4	5	6	7	8	9	10	11	12
	Print number											
1	1	2	3	4	5	6	7	11	8	9	12	10
	1	2	3	4	5	6	7	8	11	9	10	12
	1	2	3	4	5	6	7	11	9	8	12	10
2	1	2	3	4	5	6	7	10	9	12	8	11
	1	2	3	4	5	6	10	9	7	12	11	8
3	1	2	3	4	5	6	7	11	8	9	12	10
	1	2	3	4	5	6	7	11	12	10	9	8
4	1	2	3	4	5	6	7	11	12	10	9	8
	1	2	3	4	5	6	7	12	11	8	9	10

TABLE II. Illustration of method of evaluating relative quality judgments.

Score for print No. 12			Score for print No. 11		
No. of times	Position	Points	No. of times	Position	Points
8	12	96	9	12	108
13	11	143	13	11	143
12	10	120	3	10	30
13	9	117	7	9	63
2	8	16	17	8	136
2	7	14	1	7	7
Total 50		506	50		487

TABLE III. Summary of the score of the prints made from material *C*.

Final score	Print No. 1	2	3	4	5	6	7	8	9	10	11	12
Actual	50	100	150	200	272	305	393	479	481	477	487	506
Normal	50	100	150	200	250	300	350	400	450	500	550	600
Adjusted	10	21	31	41	56	63	81	98	99	98	100	104

[33] See reference 32, pp. 528–540 inclusive.

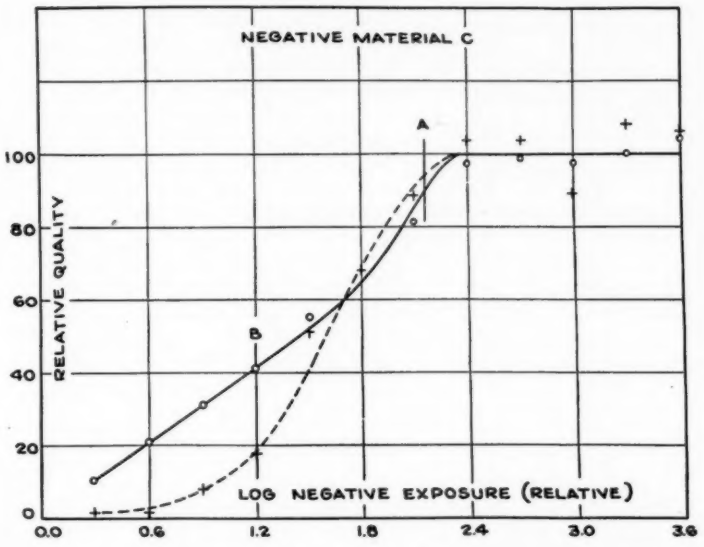

FIG. 13. Relation between print quality and negative exposure as determined statistically (solid curve) and sensitometrically (dotted curve).

terms of the magnitude and distribution of gradient errors and of brightness scale compression. This formula yields the dotted curve in Fig. 13 and results in a fairly satisfactory representation of the psychophysical quality function in the case of several negative materials, but further verification is needed and possibly modification will be required.

THE SPEED OF NEGATIVE MATERIALS

Perhaps the wisdom of spending so much time and effort to establish the relation illustrated in Fig. 13 may be questioned. Perhaps to those not intimately familiar with the problems of photography, this information appears to be of only academic interest and not worth the labor expended in its procurement. Such, however, is not the case. One immediate practical application is the establishment of a criterion in terms of which to express the speed of photographic negative materials. Many proposals have been made during the past five or six decades for expressing the speed of such products and it is recognized that at least some of these methods have not yielded results that are in agreement with practical experience.[31] Most of the suggested speed criteria have been derived from a study of the characteristic curve of the negative material, without consideration of the characteristics of the positive materials on which prints must be made. These criteria have been purely physical values derived from the sensitometric procedure. An analysis of the whole situation leads us to believe that the most rational definition of the speed of a negative material is expressed as follows: *The speed of a photographic negative material should be evaluated in terms of the minimum exposure which will yield a negative from which an excellent print can be made.* This applies specifically to the negative-positive process. For other processes, such, for instance, as reversal processes, the definition perhaps should be modified to state that the speed of the material should be evaluated in terms of the least exposure which will yield a final result (the positive obtained by reversal of materials exposed in the camera) of excellent quality. For such materials this minimum exposure can be determined by a psychophysical method similar to that just outlined and it is not likely that any purely objective method can hope to yield a result of equal significance.

It may be of interest to show the relation of the *just acceptable* and *first excellent* judgment

[31] L. A. Jones and C. N. Nelson, J. Opt. Soc. Am. **30**, 93 (1940).

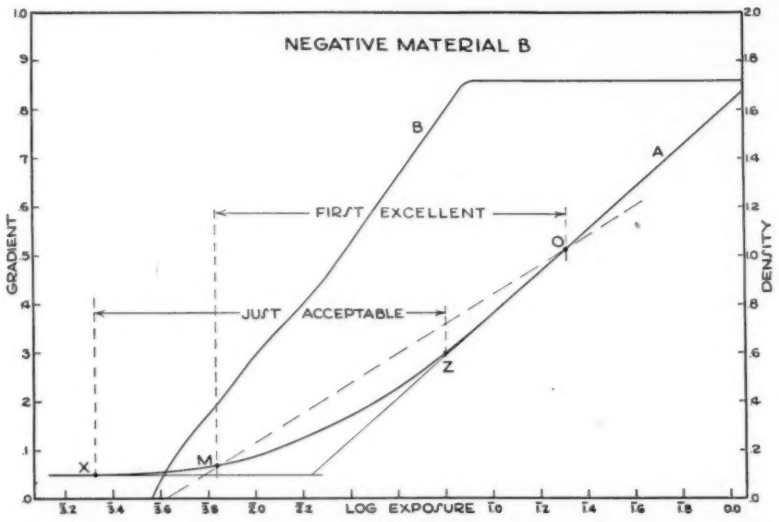

FIG. 14. Curve A, the D-log E characteristic for negative material with points indicating the portions used for making the first excellent and just acceptable prints.

results to those embodied in the negative exposure-photographic quality function. In Fig. 13 the vertical line A represents the position of the first excellent print judgment value, while the vertical line B represents the just acceptable print evaluation. It will be noted that the first excellent print corresponds closely to the beginning of the flat part of the quality-negative exposure curve. Thus the two methods agree quite well in establishing the value of minimum negative exposure yielding a reproduction of excellent quality.

But how can such long and laborious processes of psychophysical investigations be of any use for the practical problems of speed measurement and specification? Having determined for a large number of different negative materials (used in making pictures of a wide variety of exterior scenes*), the minimum negative exposure required to yield a print of excellent quality, it has been possible to establish a purely sensitometric criterion which gives results in excellent agreement with the psychophysical judgment data. This correlation is illustrated in Fig. 14. Curve A represents the usual D-log E characteristic for a negative material. The points M and O show the portion of this negative-material characteristic used when the minimum camera exposure, yielding a negative from which an excellent print can be made, is employed. It will be noted that the log exposure interval between the points M and O is 1.50 and you may wonder why this interval is used rather than the 2.20-log B_0 interval which corresponds to the average log brightness scale of outdoor scenes. The scale of image illumination incident on the negative materials in cameras is, on the average, less than that of the brightness scale of the scene being photographed because of the non-image-forming light (flare) in the camera. The average image illumination contrast for exterior scenes and average camera-lens equipment is of the order of 1 to 30 to 1 to 40, and a log exposure scale of 1.50 represents fairly well the average range of exposure incident on negative materials in the camera.

The average gradient, $\bar{G}$, of the line OM and also the gradient, $G_{\min}$, $(dD/d \log E)$ of the char-

TABLE IV. Correlation between minimum and average negative gradient.

	$\bar{G}$	$G_{\min}$	K	$\Delta K(\%)$
A	0.64	0.200	0.312	4.0
B	.61	.195	.320	6.7
C	.45	.125	.284	5.4
D	.48	.135	.282	6.0
Mean			.300	5.5

* An account of this work is as yet unpublished.

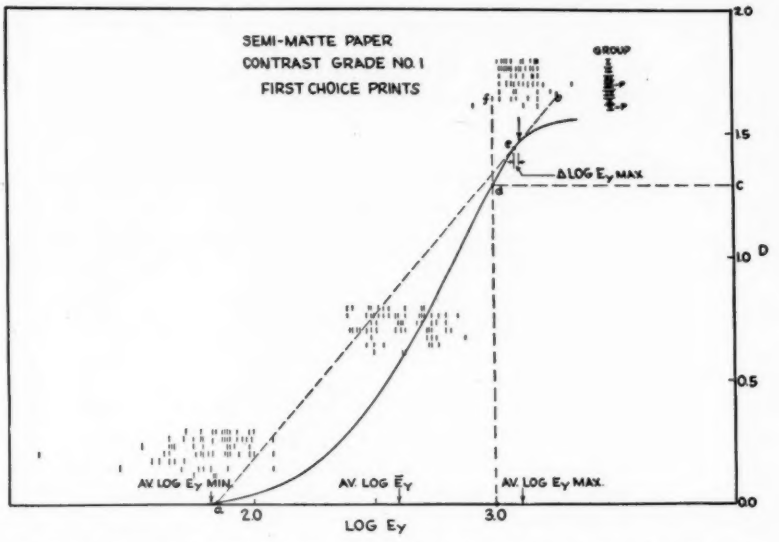

FIG. 15. Information relative to the first-choice prints made on paper grade No. 1.

acteristic curve at the point M are determined. The ratio of $G_{\min}$ to $\bar{G}$ is shown in column K of Table IV. For these four materials this ratio is practically constant. Much experimental work with many different negative materials and a large assortment of exterior scenes has further supported the constancy of this ratio. A sensitometric criterion giving excellent agreement with the psychophysical criterion can therefore be set up, this being the exposure value corresponding to a point on the D-log E characteristic of the negative material where the slope is $0.30\bar{G}$, $\bar{G}$ being measured for a log exposure interval of 1.50 extending from the point M in the direction of increasing exposure. This fractional gradient criterion has been adopted by the American Standards Association[35] for use in the definition of the speed of certain classes of photographic materials.

THE PSYCHOPHYSICS OF PRINT MAKING

Other elements in the physical stimulus process may be correlated with photographic quality yielding relationships which assist in the solution of other important photographic problems. The stimulus variable in the case previously discussed was the amount of exposure given to the negative material. Now let us consider what happens when the scene being photographed becomes the variable factor. For this purpose 130 negatives were made of as many exterior scenes varying widely in subject matter.[36] They were collected over a period of 18 months, thus including all seasons of the year, and were made at various times of the day from early morning to late afternoon. The group represents quite faithfully a representative cross section of the subject matter of which the amateur takes pictures, such as distant and semidistant landscapes and seascapes, street scenes, buildings, gardens, groups of people close to and moderately distant from the camera, flowers, trees, mountains, snow scenes, etc. All negatives received "normal" exposure (under- and overexposed negatives were carefully excluded) and all were given "normal" development. The maximum brightness ($B_{\max}$), minimum brightness ($B_{\min}$), and "average"* brightness (B_M) of each scene were measured.

[35] "American Standard method for determining photographic speed and speed number," J. Opt. Soc. Am. **33**, 479 (1943).

[36] L. A. Jones and C. N. Nelson, J. Opt. Soc. Am. **32**, 558 (1942).

* The characteristic of the scene measured was not the average brightness in the strict sense of the word. This measurement was made with a photoelectric exposure meter of the integrating type, the response of which is dependent upon the particular characteristics of the sensitive receiver and indicating electrical instrument. Perhaps the value is better characterized as integrated brightness rather than average brightness.

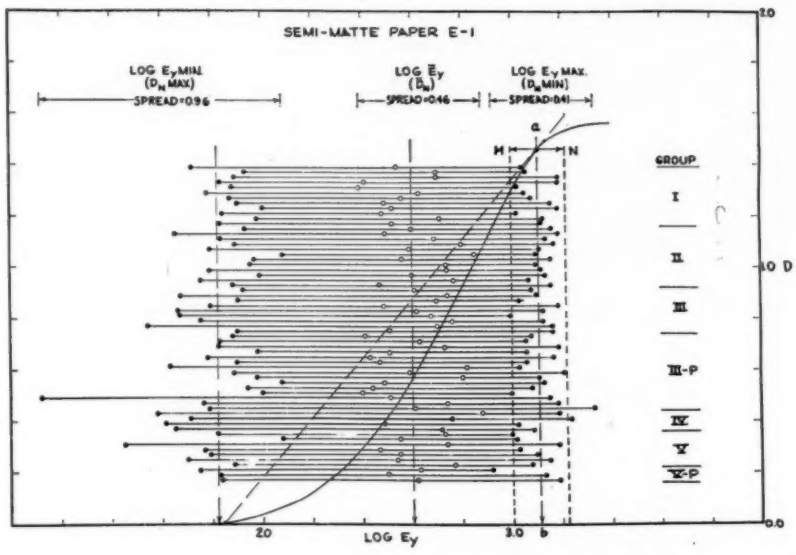

FIG. 16. Illustrating the rendition of the scene brightness scale by each of the first-choice prints made on paper grade No. 1.

At one end of each sheet of negative material, in a space not occupied by the pictures, a series of graded and known exposures (sensitometric strip) was impressed before the development of the negative. Thus the D-log E characteristic of the material applicable to the particular processing conditions employed was obtained. Three simple, unique, and measurable characteristics of each negative were then determined, namely, maximum density (D_{max}), minimum density (D_{min}), and average density (D_{Av}). Prints from each negative were made as illustrated in Fig. 9. Briefly, the variable in each of the rows is printing exposure. The different rows represent prints made on various "contrast" grades of positive material. A series of known and graded exposures was impressed on each sheet of positive material, prior to development, and from this developed sensitometric strip the density-log exposure characteristic was determined in the usual manner. In this way about 130 groups of prints, each consisting of from 15 to 25 members, were obtained.[36]

Now the judgment of these prints begins. Each observer, as before, is asked to choose the best print in each row, thus establishing his judgment of the best value of printing exposure. He is then instructed to compare the chosen prints from each row with each other and to select the grade resulting in the best possible photographic quality. In the particular case shown in Fig. 9, the observer expressed his preference for a print halfway between B and C in the upper row, for print C in the second row, and for print B in the lower row. Then, by comparing these prints he chose grade No. 2 as yielding the print of best possible photographic quality. Again, this phrase "best possible" refers to the quality obtainable with the particular negative material and the particular group of positive materials being used. Perhaps it would be better to refer to this as the best possible *relative* photographic quality. The magnitude of the judgment task can be appreciated when it is realized that each observer is required to make from 900 to 1000 separate judgments and since a group of about 30 observers was used, about 30,000 choices of best print quality had to be made. This finally resulted in a group of 130 prints, one for each negative. Measurements of maximum, minimum, and average density were then made on each print.

One method of showing these results is illustrated in Fig. 15. The sensitometric curve is that

for the No. 1 grade of the positive material. This is derived from an average of all of the sensitometric curves obtained from each of the first-choice prints on this particular positive material. The short vertical lines just above the shoulder of the characteristic curve were located as follows: the D_p max. for a given print is located at c on the density scale at the right. A horizontal line intersects the characteristic curve at d. A perpendicular dropped from this point to the log E_y scale gives the value of log E_y max. for that print. A vertical line through the point d establishes the position of the short vertical lines f. These vertical lines therefore show where the scene element of minimum brightness (minimum negative density) is rendered on the curve of the positive material. In a similar manner, the vertical lines just over the "toe" of the curve show where the scene elements of maximum brightness (maximum negative density) are rendered, and the group between shows where the scene elements of average negative density are rendered. The striking thing about this is the close grouping, with respect to the log E_y values, of the small lines representing the scene elements of minimum brightness, that is the extreme shadow region. Variation in the values of log E_y for the points derived from the average negative density (related to the "average" scene brightness) is appreciably greater, and that of the points derived from the maximum negative density (the extreme highlight of the scene) is much greater than those plotted from the minimum negative density values.

Figure 16 shows the rendering of each scene. The horizontal lines terminated by black dots show the position of the brightness scale of the object relative to the characteristic curve of the positive material for the prints of best quality. Similar graphic representations can be made for each of the "contrast" grades which must be used for scene groups of greater or lesser average brightness scales. This information now affords the answers to the following three questions:

1. What unique and measurable characteristic of a negative is the best criterion of correct printing exposure?

The answer to this is, the *minimum negative density*, that is, the density corresponding to the scene element of minimum brightness, the extreme shadow region.

2. What part of the total available exposure scale of a positive material is useful for making a print?

For the No. 1 grade shown in Fig. 16, the answer is given by the distance between the vertical line *ab* and the vertical line at the extreme toe of the curve.

3. What must be the relation between the density scale of a negative and the useful exposure scale of the positive material to produce the best possible print quality?

The answer to this question involves an analysis of the data from all of the first-choice prints and the detailed operations are too extensive to warrant discussion at this time, but the result is shown in Fig. 17. The apices of the triangles represent the useful exposure scales of the six positive materials used in this work. The

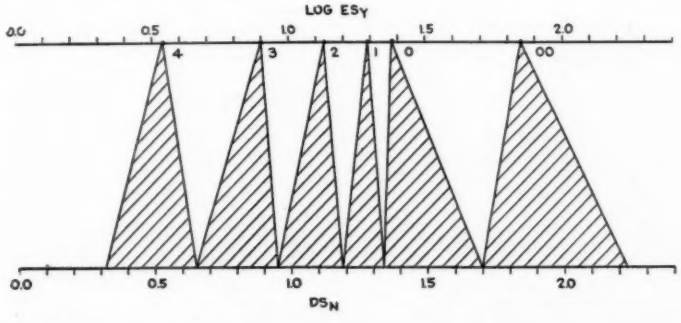

FIG. 17. Summary of the relation between negative density scale and paper log exposure scale.

bases of these triangles embrace the ranges of negative density scale which resulted in best possible print quality on the several grades.

CONCLUSION

The procedures which have been followed in this work may appear to bear very little relationship to the classical methods of psychophysical investigation with which many of this audience are familiar. They can, however, be reduced in principle to one of the three methods set forth by Fechner. But it is impossible to discuss this in detail on this occasion. It may be of interest to return now to our "map" (Fig. 1) showing the relationship between the realms of the physical, psychological, and psychophysical sciences. It is apparent that the relationships which we have discussed fall without doubt in the psychophysical category. The photographic prints on which our observers have passed quality judgments correspond roughly to 3. They are illuminated surfaces on which there is an extremely complicated array of visual field elements varying in brightness. The configuration of these elements is such that they represent certain objects with which the observer is more or less familiar by virtue of his experience. An image of this picture is formed on the retina of the eye and sensory nerve impulses travel inward along the optic nerve and finally, after traversing various non-cortical synaptic regions of the cerebrum, arrive in the sensory projection areas of the cerebral cortex and here give rise to visual sensation. The observer's judgment of print quality, however, is not conditioned entirely by the immediate deliverances of the visual sense organ. The nerve impulses are propagated further into the association areas and there is little doubt that the psychical response which conditions the judgment of print quality involves perception and possibly even more complicated modes of mental functioning. Perhaps it is not possible to decide with certainty what activities are involved on the psychical side of the psychophysical relationship, although the physical aspect can be very intimately identified with the object 3 in the physical realm. There is little justification for the position adopted by some that the science of psychophysics is limited to relationships between physical factors and *sensory* response. Commenting on Fechner's definition of psychophysics, Titchener points out that as thus defined the sphere of psychophysics is as broad as that of psychology and that consequently there will be a psychophysics of sensation, of perception, of feeling, of action, of attention, etc.

While it may be interesting to speculate on the mental processes which are involved in the final judgment of relative print quality, the outcome of this speculation affects in no way the usefulness of the psychophysical relations which have been established, since these relations do give us very definite and unequivocal answers to questions which cannot possibly be answered by the consideration of physical factors alone. It is hopeless to attempt from any *a priori* applications of objective tone reproduction theory to obtain adequate solutions of the problems of photographic quality; solutions which are valid and not subject to possible contradiction by direct experiential evidence.

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Schmidt Systems as Spectrograph Cameras*

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Small chromatic aberrations and large field of good definition make Schmidt systems particularly suited for spectrograph cameras. The outstanding disadvantages—spherical focal surface and loss of light due to the position of the plate in front of the mirror—play different roles in cameras of large and small focal lengths. For focal lengths above 30 inches, photographic plates on thin glass can be bent to the necessary curvature without great risk of breakage. For cameras of shorter focal length, curvatures necessitating the use of film can be avoided by the introduction of a field-flattening lens. A plano-convex lens placed close to the focus has proved completely satisfactory for cameras of relative apertures $f/3$ and $f/1.5$. The use of flat plates without field flattener restricts the field of good definition to an angular extent of $114.6(d/A)^{\frac{1}{2}}$ degrees (d permissible diameter of the circle of diffusion, A diameter of the collimator); this is sufficient for many purposes. The loss of light due to occulting by the plate can be avoided by an off-axis construction. This is completely satisfactory for cameras of large focal length where the plate can be bent to the curvature of the focal surface. Field-flattening lenses are usable, but the aberrations introduced by them will be those corresponding to the normal system of larger aperture of which the off-axis system is part. Consequently, the aberrations will be relatively large and the usefulness of field-flattening lenses with the off-axis construction will be restricted to relative apertures of $f/4$ and smaller. The use of flat plates in off-axis systems results in a sidewise curvature of the spectrum on the plate with a radius of curvature kf^2/A; the factor k, depending on details of the construction, will usually be between 1 and 1.5. Such a curvature is objectionable. As the use of film generally reduces the accuracy of wave-length measures and is, therefore, equally undesirable, off-axis systems of short focal lengths are of restricted usefulness. For cameras of very short focal lengths, the small scale of the spectrum permits the use of small plates, thus minimizing the loss of light in a normal system, but a range of focal lengths below 30 inches will always remain for which the full advantages of Schmidt systems are not available without some loss of light.

SMALL chromatic aberrations and large field of good definition make Schmidt systems particularly suited for spectrograph cameras. The basic idea of B. Schmidt[1] to remove the coma of a spherical mirror by a diaphragm in the center of curvature and the spherical aberration by a weak aspherical lens (Fig. 1a) has proven its value in numerous telescopes and spectrograph cameras. The Schmidt system is so well known by now that it is hardly necessary to emphasize that the definition obtained with a well-made Schmidt system is unsurpassed. To utilize the small

* Paper presented at the Twenty-Eighth Annual Meeting of the Optical Society of America, Pittsburgh, Pennsylvania, October 7–9, 1943.

[1] B. Schmidt, Mitt. Hamb. Sternwarte in Bergedorf **7**, No. 36 (1931).

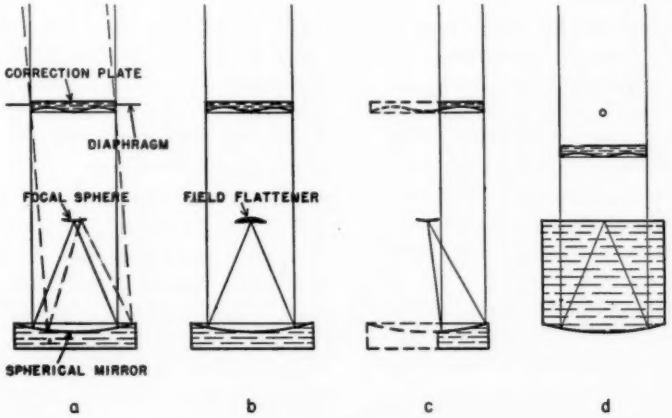

FIG. 1. Types of Schmidt cameras.

chromatic aberrations to the fullest extent in a spectrograph, a collimator well corrected for color is, of course, necessary. If an off-axis parabolic mirror is used as collimator, the spectrograph is free from chromatic aberrations to such an extent that no change of focus is necessary to observe widely different wave-length regions. This would be equally true for an autocollimating spectrograph with a Schmidt system.

Some disadvantages are inherent in the construction of the system. The role which they play depends largely on individual requirements. For instance, the large size of the mirror necessary to obtain a large unvignetted field and the over-all camera length of twice the focal length may be objectionable for a telescope, but will, in general, not present serious difficulties for a spectrograph camera. The two outstanding disadvantages are the spherical focal surface and the loss of light due to the position of the plate in front of the mirror. The question whether in any given case the Schmidt system is preferable to a lens system is essentially identical with the question to what degree these disadvantages may be neutralized.

In a discussion of this question, it is well to remember that the entrance pupil of a spectrograph camera is limited by the size of the dispersing element. Consequently, a camera of large focal length will always have a small relative aperture, and the relative aperture of a spectrograph camera will in many instruments be much smaller than that of the average Schmidt telescope. In many spectrograph cameras, it may, therefore, become possible to omit the correcting plate. The spherical aberration remains then uncorrected; but, as Schmidt has already emphasized, the disk of diffusion for a spherical mirror with relative aperture $f/10$ is only 6.4″ in the paraxial image point, decreasing with the third power of the relative aperture, and the minimum size is only one-quarter of this value. For relative apertures smaller than $f/10$, this disk of diffusion will frequently be smaller than the resolving power of the photographic plate. Thus, if it becomes possible to omit the correcting plate, the Schmidt system is reduced to a spherical mirror and a diaphragm, certainly one of the simplest optical systems.

The curvature of the focal surface presents no difficulties at all if the use of film is permissible, since cylindrical curvature in the direction of dispersion is quite sufficient in a spectrograph camera. For focal lengths above 30 inches, photographic plates on thin glass can be bent without great risk of breakage. But the most satisfactory way is to avoid the difficulty entirely by the use of a field-flattening lens. That a plano-convex lens, placed close to the focus with the plane side towards the photographic plate (Fig. 1b), may be used to remove the curvature of the focal surface was mentioned by Schmidt, but has been widely overlooked. The focal length of this lens is, approximately, that of the mirror divided by the index of refraction of the material of the lens.

The range of usefulness of the field-flattening lens is well illustrated by the computations by F. E. Ross[2] for the 48-inch Schmidt telescope for Mt. Palomar. The result, that the plano-convex lens should still be satisfactory for a focal length of 120 inches at a relative aperture $f/2.5$, permits the conclusion that the additional aberrations introduced by the field-flattening lens will be negligible for all spectrograph cameras with relative aperture $f/2.5$ or smaller. That field-flattening lenses perform excellently at even larger relative apertures, is shown by experience with Schmidt cameras at the Mt. Wilson Observatory. Plano-convex field-flattening lenses have been added in a camera of 6 inches focal length, relative aperture $f/3$, and of 3 inches focal length, relative aperture $f/1.5$. No impairment of the excellent definition of these cameras can be detected at a distance of 5° from the optical axis, i.e., at the edge of the field actually used. Experience with these cameras indicates that the adjustment of the field-flattening lens is not critical. The lenses are mounted in the plate-holders; the combination can be moved to obtain several spectra on one plate without noticeable effect on the definition.

It may be concluded that, for the great majority of conditions, the plano-convex field-flattening lens will be entirely satisfactory. However, for cameras of large relative aperture the high order aberrations of the field flattener will increase rapidly. Trials have shown that for a camera of 6 inches focal length and relative aperture $f/0.9$ the field flattener destroys the

[2] F. E. Ross, Astrophys. J. **92**, 400 (1940).

definition. The use of more complicated field flatteners may extend their usefulness, but it seems evident that for very large relative apertures the elimination of the curvature without loss of definition will become impossible.

The introduction of the field-flattening lens transforms the Schmidt system into a system which, in some respects, is similar to the two mirror systems with correcting plate proposed by J. G. Baker.[3] Obviously, no fundamental difference exists between removal of the curvature by a lens or its removal by a mirror. In view of the satisfactory performance of the lens, a mirror seems to offer no practical advantage as far as removal of curvature is concerned. The main advantage of Baker's systems is their smaller length. Whether this outweighs the disadvantages —greater difficulty of manufacturing and greater loss of light due to occulting by the second mirror—will depend on individual requirements.

The use of flat plates without field flattener restricts the field of good definition. The best compromise focus in this case is, obviously, that for which the extrafocal image in the center and the intrafocal image at the edge of the field have equal size. Simple geometrical considerations show that the distance from the center to the intersection of plate and focal surface is then approximately three-tenths of the diameter of the field. If d is the permissible diameter of the out-of-focus image and A the diameter of the diaphragm, which equals that of the collimator in a spectrograph, the usable field has a radius of $114.6(d/A)^{\frac{1}{2}}$ degrees. This is sufficient for many purposes.

For cameras of very large relative aperture, where the use of flat plates may be unavoidable, a larger usable field can be obtained by a solid mirror system. In its ideal form, this consists of one piece of glass with the correction surface at one end and the internally reflecting mirror at the other. The glass now plays the role of an immersion liquid. Depth of focus and curvature of the field are determined by the geometrical focal length; scale and speed by the smaller effective focal length. The usable field on a flat plate is extended by a factor approximately equal to the index of refraction of the material of the solid mirror.

The idea of the solid-glass camera was conceived, but not mentioned publicly, by Schmidt. Later, Sinclair Smith rediscovered this type of system, and his discovery was made public by Hendrix and Christie.[4] Baker,[5] the third independent discoverer, has carried out the theoretical investigation of the system. The solid system is difficult to make and has a focus which is not easily accessible. It is not known that any of the modifications which make the focus easily accessible, such as the folded type of Hendrix and Christie and the very interesting off-axis type of Baker, has been built and used.

Closely related to the solid-glass camera, and sometimes included in this classification, is the thick-mirror type. This consists (Fig. 1d) of a piece of glass with the focal surface at one end and the internally reflecting mirror at the other and of a separate correcting plate, whose best position is somewhat closer to the mirror than the center of curvature. This construction, again known to Schmidt but not made public, was first described by Hendrix.[6] The thick-mirror system is a compromise solution, optically inferior to either the solid or the ordinary Schmidt system. But the remaining aberrations are small enough to permit the use of very large relative apertures. A camera of this type, with a geometrical focal length of two inches, a geometrical relative aperture $f/0.95$, and an effective relative aperature $f/0.65$, is in use at the Mt. Wilson Observatory. The definition is practically perfect. Even larger effective relative apertures seem accessible with systems of this type.

The loss of light due to occulting by the photographic plate can be avoided by an off-axis construction (Fig. 1c) which was first described by Dunham.[7] This is completely satisfactory for cameras of large focal length where the plate can be bent to the curvature of the focal surface. Cameras of this type, with focal lengths of 32 inches and 72 inches, are in regular use in the Coudé spectrograph at the 100-inch telescope. If

[3] J. G. Baker, Proc. Am. Phil. Soc. **82**, 339 (1940).

[4] D. O. Hendrix and W. H. Christie, Sci. Am. **161**, 118 (1939).

[5] J. G. Baker, Proc. Am. Phil. Soc. **82**, 323 (1940).

[6] D. O. Hendrix, Pub. Astronom. Soc. Pac. **51**, 158 (1939).

[7] Theodore Dunham, Jr., Pub. Astronom. Soc. Pac. **8**, 110 (1935).

the relative aperture is sufficiently small, the correcting plate may be omitted in an off-axis system as well as in the ordinary system. The camera with 112 inches focal length of the Coudé spectrograph at the 100-inch telescope is of this type. Cameras of this type will be just as satisfactory in most spectrographs with high linear dispersion.

Field-flattening lenses can, of course, be used in off-axis systems. But the aberrations introduced by them will be close to those which occur in the normal system of larger relative aperture of which the off-axis system is part. Consequently, the aberrations are relatively large and the usefulness of field-flattening lenses will be

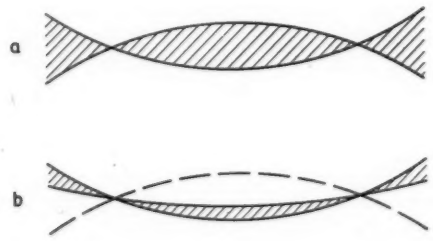

FIG. 2. Sidewise curvature of spectrum on a flat plate with off-axis Schmidt camera.

restricted to relative apertures of about $f/4$ and smaller.

The use of flat plates without field flattener in an off-axis system results in a sidewise curvature of the spectrum. On a flat plate, the spectrum is in focus only where the plate intersects the focal surface. The spectrum of one element of the slit is, therefore, not a line, but covers an area included between two curves whose distance, in any point, equals the size of the out of focus image. For a normal Schmidt system (Fig. 2a), the two curves are symmetrical with respect to the center line which connects the two points where the spectrum is in focus. In the off-axis system (Fig. 2b), part of this area is missing and the remaining part is asymmetrical to the center line of the normal system, enclosed between two curves concave towards the centerline near its middle. The radius of curvature of the spectrum is, approximately, kf^2/A, where f is the focal length of the camera, the factor $k=2A/(A_0-A)$, and A_0 the aperture of the normal system from which the off-axis system with the aperture A is derived. The value of k will usually be between 1 and 1.5. The sidewise curvature may be objectionable. As the use of film generally reduces the accuracy of wave-length measures and is, therefore, not always permissible, off-axis systems of relative aperture too large to permit the use of a field-flattening lens and of a focal length too short to permit bending of the photographic plate are of restricted usefulness.

For cameras of very short focal length, the small scale of the spectrum permits the use of small plates. The loss of light in a normal system can thus be minimized. In a spectrograph attached to a reflecting telescope, loss of light may be avoided if the plate is inside the part of the beam obscured by a secondary mirror. But, in general, a range of short focal lengths will always remain for which the full advantages of Schmidt systems are not available without some loss of light.

In those systems of low relative aperture where the correcting plate may be omitted the spectral range accessible is not limited by absorption losses, but merely by the reflecting power of the mirror. Systems of this type could probably be used to good advantage in many spectrographs for general spectroscopic purposes. Absorption losses in the correcting plate, which is always thin, may be minimized by proper choice of material and will always be smaller than in a lens system with equal relative aperture. The thick-mirror type, made from glass with high transparency, has smaller absorption losses than the known lens systems of very high relative aperture which, in addition, are definitely inferior in definition.

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Area in Color Harmony*

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1. INTRODUCTION

A PREVIOUS paper[1] gave a geometric formulation of classical color harmony. It purposely ignored the effect of relative area of the color patches. Such a procedure has the advantage of simplicity and is in accordance with previous work of color experts. The present paper, however, extends the theory by taking into account the effect of area. The colors are chosen by classical methods; but they are now made even more pleasing by the adjustment of area, which increases the aesthetic effect by bringing in additional elements of order.

The effect of area in color harmony has been hardly more than mentioned in the literature. All artists know that large areas of low chroma can be balanced by small areas of high chroma, but apparently this principle has scarcely advanced beyond the qualitative stage. For instance, Ostwald[2] states that "*Je kleiner, um so reiner,*" but he does not apply his maxim. Indeed, he seems to feel that any such relation concerns chroma only and cannot be applied to brightness variations.

Experiments made by the authors with color chips showed that relative area is a function of both chroma and value. It was found that a pleasing balance can be obtained when the product of each area and its distance from the adaptation point is the same in ω space. The principle is not limited to variation in chroma but applies to any direction in the 3-dimensional space. Other pleasing effects are obtained when the scalar moments in ω space are simple multiples of each other. The balance of scalar moments appears to be a new principle of color harmony—a principle of wide applicability and practical importance. Its general use with any state of visual adaptation will be considered in a subsequent paper; but in the present treatment, only the special case of adaptation to a gray of Munsell value 5 will be considered.

The moments in colorspace, which play such an important part in the new theory, are treated most simply by means of *weighted points*. The mathematics of the subject is developed, and the calculated areas for harmonious effects are tabulated for a large number of examples.

Another concept is the *balance point* of a system of color patches. Unlike the calculation of scalar moments, the determination of the balance point is carried out in x space instead of ω space. The balance point is important because it gives a measure of the over-all color effect of the design. By choice of a proper balance point, the artist can emphasize a desired psychological effect.

2. POSTULATES

The discussion of the effect of area in color harmony is based on three postulates:

(1) A pleasing balance among n color patches is obtained when the scalar moments about the adaptation point in ω space are equal for all the patches.

(2) Other pleasing balances are obtained when the scalar moments of the various color patches are simple multiples of each other.

(3) The psychological effect of a set of color patches depends on the balance point obtained in x space.

By the *scalar moment* of a color patch is meant the product of the area of the patch and the distance in ω space between the adaptation point and the color point. The *adaptation point* is the point in ω space corresponding to the state of adaptation of the eye. The *balance point* of a set of color patches is defined as the point corresponding to the color obtained by disk mixture of the component colors, the areas on

* Paper presented at the Twenty-Eighth Annual Meeting of the Optical Society of America, Pittsburgh, Pennsylvania, October 7–9, 1943.

[1] P. Moon and D. E. Spencer, J. Opt. Soc. Am. **34**, 46 (1944).

[2] W. Ostwald, *Die Harmonie der Farben* (Leipzig, 1922); *Farbkunde* (Leipzig, 1923).

the disk being directly proportional to the areas of the corresponding color patches. The same result can be obtained by viewing the design from a sufficient distance, also by simple analytical methods.

The postulates are of great generality, since they apply for any state of adaptation. Two outstanding special cases are of interest, however:

(a) Small color patches are viewed against a large, uniform background. The color patches are so small that the eye adapts itself to the background.

(b) The color patches are large and the eye is focused on them so that it adapts itself to a color represented by the balance point. Such a condition may obtain when one studies a painting or a poster. Obviously there are infinitely many other conditions of adaptation, all of which are covered by the postulates. In the present paper, however, attention will be confined exclusively to (a) with a neutral gray background having a Munsell value of 5.

TABLE I. Some experimental results on area balance. Neutral background (N 5).

Sample No. 1	Area	Sample No. 2	Area	Calculated ratio
BG 4/4	1.0	*BG* 2/4	0.4	0.37
PB 7/4	0.5	*PB* 6/4	1.0	0.54
PB 7/4	0.2	*PB* 5/4	1.0	0.24
PB 7/4	0.5	*PB* 4/4	1.0	0.54
PB 6/4	1.0	*PB* 4/4	1.0	1.00
PB 6/4	0.6	*PB* 3/4	1.0	0.54
PB 6/4	0.3	*PB* 2/4	1.0	0.37
YR 8/2	0.7	*YR* 7/2	1.0	0.79
YR 8/2	0.6	*YR* 6/2	1.0	0.62
YR 8/2	0.55	*YR* 5/2	1.0	0.56
YR 8/2	0.8	*YR* 3/2	1.0	0.79
YR 8/2	1.0	*YR* 2/2	1.0	1.00
BG 8/2	0.8	*BG* 7/2	1.0	0.78
BG 8/2	0.6	*BG* 4/2	1.0	0.62
BG 8/2	0.8	*BG* 3/2	1.0	0.78
BG 8/2	1.0	*BG* 2/2	1.0	1.00

Postulates 1 and 2 were suggested by the results of experiments. Munsell color chips 12×17 mm were placed on a large neutral background (N 5) and were viewed at a distance of 35 cm. Illumination of approximately 1000 lumens/m² was obtained from skylight. The eyes were adapted to the neutral background, rapid eye movements were used, and tests indicated that the adaptation was not altered appreciably by the presence of the color samples.

Two color chips were employed in any one test, and the observer slid one over the other until the exposed areas seemed to have equal prominence. The relative areas were then measured. Successive trials of the same pair of samples indicated that the results could be duplicated to within approximately ± 0.1 of the larger area. Two observers, both of whom appeared to have normal color vision, obtained the data; and their individual findings ordinarily checked within the above limits.

Table I lists a representative set of data for one observer. The first and second columns give the Munsell specification for one sample and its relative area. The third and fourth columns give similar information for the other sample. The final column lists the ratio of the two areas, calculated by means of scalar moments in ω space. The calculated ratios should check the lower of the two relative areas, and the table shows in most cases a fairly good approximation.

Tests were made for variations in value, chroma, and hue. White and black backgrounds were employed, in addition to the gray background. In all cases, a satisfactory correlation existed between experiment and theory. It is possible that a more elaborate experimental investigation, with results analyzed statistically by the methods of modern experimental psychology, might be of value; though the present work appears to be sufficient to establish the practical validity of the method of scalar moments in color harmony.

In these experiments, the observers adjusted the areas until the two color patches appeared to have equal prominence. This arrangement gave a pleasing sensation of balance. It was found that other pleasing effects could be obtained by making the scalar moments simple multiples of each other. For example, if two color chips give a sensation of balance when their areas are in the ratio of 0.5 : 1, they will also be pleasing in the ratios 0.16 : 1, 0.25 : 1, 1 : 1, 1 : 1.5. In the cases mentioned, the scalar moments have the ratios 1 : 1, 1 : 3, 1 : 2, 2 : 1, 3 : 1. The sensation produced by unequal moments is quite different from that produced when the moments are equal, for it accents one or the other chip, whichever has the higher moment. Moment ratios of more than three, however, are hard to recognize and are of doubtful importance.

3. SCALAR MOMENTS IN COLORSPACE

The mathematical formulation of color harmony described in this series of papers is based on the C.I.E. color 3-space into which a metric has been introduced (ω space).[3] The characteristic of this space is that an equal distance in any direction and in any part of the space represents an equal perceptual step. The ω space can be applied with any state of visual adaptation, but only adaptation to a gray of Munsell value 5 will be considered in the present paper.

Figure 1 shows the rectangular coordinate system (ω^1, ω^2, ω^3) as well as the cylindrical coordinate system (r, θ, z). The z axis represents the achromatic colors, while the planes of constant hue are arranged at various angles θ about this axis. The adaptation point A is on the neutral axis. Any color is represented by a point in the 3-space, as $P(r, \theta, z)$ in Fig. 1. Given two colors, P and Q, of the same hue and having a helios equal to that of the background to which the eye is adapted, then P, Q, and A are on a horizontal line. If the color patch P has area S_1, and Q has area S_2, the scalar moments about A are evidently S_1r_1 and S_2r_2, respectively. It is well known that if the chroma of Q is higher than that of P, then Q should have the smaller area. According to Postulate 1, a pleasing sense of balance is obtained if the moments are equal, or

$$S_1r_1 = S_2r_2. \tag{1}$$

But P and Q need not be on a horizontal line, nor need they be in the same plane. For example, the moments for P' and Q' of Fig. 1 are

$$S_1[(r_1)^2+(z_1-z_A)^2]^{\frac{1}{2}} \quad \text{and} \quad S_2[(r_2)^2+(z_2-z_A)^2]^{\frac{1}{2}}.$$

A pleasing result is obtained when the areas are adjusted so that

$$S_1[(r_1)^2+(z_1-z_A)^2]^{\frac{1}{2}} = S_2[(r_2)^2+(z_2-z_A)^2]^{\frac{1}{2}}. \tag{2}$$

According to Postulate 2, other pleasing effects are obtained by use of moments that are simple multiples of each other. Thus if it is desired to accent P,

$$nS_1[(r_1)^2+(z_1-z_A)^2]^{\frac{1}{2}} = S_2[(r_2)^2+(z_2-z_A)^2]^{\frac{1}{2}} \tag{3}$$

where n is an integer greater than unity. Similarly, if Q is to be given greater prominence, the n is placed on the right of Eq. (3). With any number m of color patches, Postulates 1 and 2 show that pleasing effects are obtained when the following relationship is satisfied:

$$\begin{aligned} n_1S_1[(r_1)^2+(z_1-z_A)^2]^{\frac{1}{2}} &= n_2S_2[(r_2)^2+(z_2-z_A)^2]^{\frac{1}{2}} = \cdots \\ &= n_mS_m[(r_m)^2+(z_m-z_A)^2]^{\frac{1}{2}}, \end{aligned} \tag{4}$$

where S_1, S_2, $\cdots S_m$ are the areas and n_1, n_2, $\cdots n_m$ are 1, 2, or 3.

The same conclusions can be expressed in the Munsell notation, for r in ω space corresponds approximately to Munsell chroma, θ corresponds to hue, and z corresponds to value.[3] Munsell chroma is taken equal to the number of units of r from the neutral axis; Munsell value is taken as $z/8$, in accordance with the findings of Bellamy and Newhall.[4] The scalar moment of any color point about the adaptation point N 5 is then

$$S[(\text{chroma})^2+64(\text{value}-5)^2]^{\frac{1}{2}} = \text{area} \times \text{moment arm}. \tag{5}$$

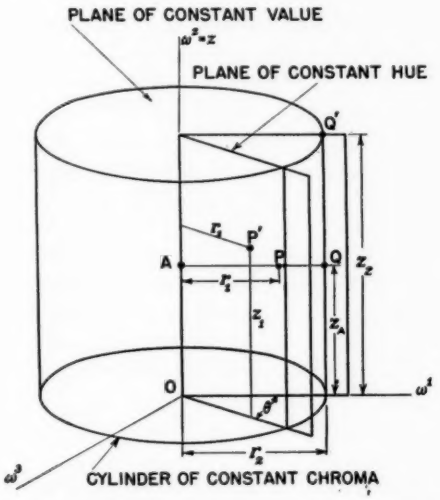

FIG. 1. Coordinate systems in ω space. P and Q represent two colors of the same Munsell value and hue, P' and Q' two colors which differ in value and hue.

[3] P. Moon and D. E. Spencer, J. Opt. Soc. Am. **33**, 270 (1943).

[4] B. R. Bellamy and S. M. Newhall, J. Opt. Soc. Am. **32**, 465 (1942).

TABLE II. Moment arms about N 5.

Munsell value	Chroma/0	Moment arms for /2	/4	/6	/8	/10	/12	/14
0 and 10	40	—	—	—	—	—	—	—
1 and 9	32	32.1	32.4	32.6	33.0	33.6	34.2	35.0
2 and 8	24	24.1	24.4	24.8	25.3	26.0	26.8	27.8
3 and 7	16	16.1	16.5	17.1	17.9	18.9	20.0	21.3
4 and 6	8	8.25	8.94	10.0	11.3	12.8	14.4	16.1
5	0	2	4	6	8	10	12	14

It is convenient to have the moment arms tabulated for easy reference, and the computed results are given in Table II. For example, suppose that the two colors P 6/8 and P 3/8 are chosen for a harmony. What should the relative areas be? Calculation (Table II) shows that the moment arms are 11.31 and 17.89, respectively. Thus

$$(\text{Area of } P\ 3/8)/(\text{Area of } P\ 6/8) = 11.31/17.89 = 0.63.$$

Other harmonious ratios of areas are 0.31, 0.21, 1.26, 1.89, according to Postulate 2.

4. THE BALANCE POINT

The preceding section deals with scalar moments in ω space and their relation, in accordance with Postulates 1 and 2. Now consider the third postulate, which is concerned with the balance point of the color ensemble. Field[5] in 1845 stated that a set of color patches would be harmonious if it balanced to a neutral gray. Munsell[6] is credited with a similar statement, though he certainly did not limit his harmonies to this very circumscribed class. The balance point need not be achromatic but can be placed wherever the artist wishes.

If two colors to be used in a design are placed on a Maxwell disk and the two areas are made directly proportional to the areas of the design, the disk mixture will have the trichromatic coordinates of the resulting balance point. Or, if the design is viewed at sufficient distance so that its individual elements cannot be resolved by the eye, the visual effect will represent the balance point. Let the trichromatic coordinates of the two colors be (X_1, Y_1, Z_1) and (X_2, Y_2, Z_2) and their areas be S_1 and S_2. The coordinates of the balance point will then be

$$\begin{aligned} X &= [(X_1S_1)+(X_2S_2)]/(S_1+S_2), \\ Y &= [(Y_1S_1)+(Y_2S_2)]/(S_1+S_2), \\ Z &= [(Z_1S_1)+(Z_2S_2)]/(S_1+S_2). \end{aligned} \tag{6}$$

The position of the balance point in the chromaticity diagram is obtained by calculating x and y in the usual manner:

$$\begin{aligned} x &= X/(X+Y+Z), \\ y &= Y/(X+Y+Z). \end{aligned} \tag{7}$$

The balance point for more than two colors is handled by an obvious extension of these equations.

Evidently Eqs. (6) and (7) may represent a gray as advocated by Field; or they may be chromatic. The general effect of the design is quite different in the two cases, and this fact constitutes the reason for introducing the concept of the balance point. If the designer wishes to obtain a cool and restful effect, he will naturally choose such colors that the balance point is in the greens; while if he wishes a warm and stimulating effect, he will use a balance point in red or orange.

Table III gives an attempt at a definite statement of the psychological effect of the balance point. When the Munsell chroma is greater than 5, decided differences in effect are obtained by varying the hue associated with the balance point. For chroma less than 5, however, hue has little effect and there is a tendency toward neutrality. For a balance point whose Munsell value is above 6.5, the design tends to

TABLE III. Psychological effect of the balance point.

Munsell hue	Munsell value	Munsell chroma	Psychological effect
R		>5	Very stimulating, very warm
YR		>5	Stimulating, warm
Y		>5	Slightly stimulating, slightly warm
GY		>5	Slightly restful, neutral temperature effect
G		>5	Restful, slightly cool
BG		>5	Very restful, very cool
B		>5	Non-stimulating, cool
PB		>5	Non-stimulating, cold
P		>5	Slightly stimulating, neutral temperature effect
RP		>5	Stimulating, slightly warm
Any	>6.5	Any	Cheerful
Any	<3.5	Any	Gloomy
Any	Any	<5	Non-stimulating, neutral temperature effect

[5] George Field, *Chromatics* (London, 1845).
[6] A. H. Munsell, *A Color Notation* (Munsell Color Company, Baltimore, 1941).

be cheerful; while if below 3.5, the effect is ordinarily gloomy (Fig. 2). Such characteristics are known and used by all artists, but it is believed that they are here tied in for the first time with scientific color specification.

5. WEIGHTED POINTS IN COLORSPACE

Sections 3 and 4 have dealt with the calculation of area in color harmony. For a fixed state of adaptation, the computations are very easy and the simplest mathematics suffice. But the subject can be handled in an elegant manner by use of the concept of the *weighted point*, and this concept applies not only to the subject matter of the present paper but also to the more general case of variable adaptation. Thus it seems advisable to develop the mathematics of the weighted point, which can be considered as a particular type of homogeneous point coordinate in 3-space.

The concept was first developed to describe properties of mass points in mechanics, where a point of ordinary space has associated with it a scalar—the mass. In colorspace, also, each point has associated with it a scalar, which in this case is the area of the color patch.

The weighted-point idea can be introduced either into metric colorspace or into affine colorspace. The metric colorspace or ω space[3] is an ordinary Euclidean space in which distances have meaning but in which the addition of color points does not represent the mixture of colors. The ordinary colorspace or x space, on the contrary, allows addition of colors but does not give general significance to distances or angles. It will be helpful to consider weighted points in both spaces.

Consider first the metric colorspace, where a point is specified[7] by the coordinates ω^k ($k=1, 2, 3$). The corresponding weighted point can be designated[8] by Ω^κ ($\kappa=0, 1, 2, 3$). The coordinates

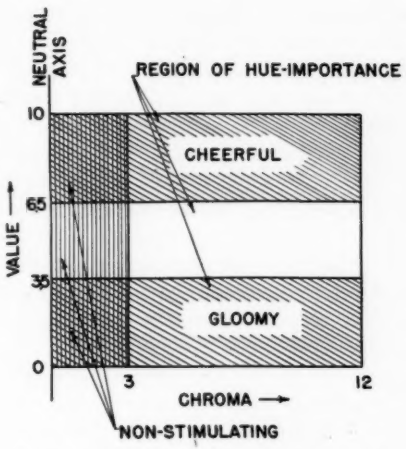

FIG. 2. A plane of constant hue (θ=const.).

Ω^1, Ω^2, and Ω^3 will specify the color ($\Omega^1=\Omega^0\omega^1$, $\Omega^2=\Omega^0\omega^2$, $\Omega^3=\Omega^0\omega^3$), while Ω^0 will designate the area of the color patch. Latin indices will be used[8] when the superscripts are to run from 1 to 3, while Greek indices will indicate the range 0, 1, 2, 3. The point coordinates ω^k and the weighted-point coordinates Ω^κ are related by the following equation:

$$\omega^k=\Omega^k/\Omega^0, \quad \text{or} \quad \Omega^k=\Omega^0\omega^k. \tag{8}$$

Similarly, a set of weighted points can be defined in the affine colorspace, in which the point coordinates are x^k. The corresponding weighted-point coordinates are called X^κ. X^1, X^2, and X^3 designate the color by the usual trichromatic method, while X^0 expresses the area. The relation between x^k and X is

$$x^k=X^k/X^0, \quad \text{or} \quad X^k=X^0x^k. \tag{9}$$

Note that $X^0=\Omega^0$.

In mechanics, moments about an axis are balanced to determine equilibrium conditions, and weighted points are added to obtain the center of gravity of a set of masses. In colorspace, it is convenient to perform somewhat similar operations. Consider the useful mechanical concept—the *moment*. The moments of mechanics are taken about an axis and are vector moments, possessing magnitude, direction, and polarization.

[7] Tensor notation will be used. See J. A. Schouten and D. J. Struik, *Einführung in die neueren Methoden der Differentialgeometrie* (P. Noordhoff, Groningen, 1935). A tensor is represented by a letter with superscript or subscript, as ω^k, g_{ij}. Powers will be distinguished by numerals outside parentheses, as $(\omega^k)^2$; and if m tensors of the same kind are used, they are distinguished by subscripts outside the parentheses, as $(\omega^k)_1$, $(\omega^k)_2$, $\cdots(\omega^k)_m$. This notation is easier on the typesetter than the placement of a label *beneath* the letter, as done by Schouten and Struik.

[8] D. E. Spencer, "A tensor interpretation of Study's *Geometrie der Dynamen*," Ph.D. thesis, M.I.T. (1942); "Geometric figures in affine space," "The tensor representation of Study's *Geometrie der Dynamen*," to be published in J. Math. Phys.; J. Frank. Inst. **236**, 293 (1943).

The moments in colorspace, on the other hand, are *scalar moments about a point*. The scalar moment is equal to a distance times an area. The distance is from a preferred point (the adaptation point) to the point in ω space representing the color; the area is the area of the color patch. Since the concept of distance has no general validity in affine colorspace, moments are evidently without meaning in this space. The ω space, however, was especially developed for color harmony,[3] and distances in this metric space are directly related to perceptual differences in color.

Let the preferred point have the coordinates a^k in ω space. Then the scalar moment of a color patch having the coordinates Ω^k will be

$$M=[g_{ij}(\Omega^0 a^i-\Omega^i)(\Omega^0 a^j-\Omega^j)]^{\frac{1}{2}}$$
$$=\Omega^0[g_{ij}(a^i-\omega^i)(a^j-\omega^j)]^{\frac{1}{2}}. \qquad (10)$$

The Einstein summation convention[7] is used in Eq. (10), and the second equation is obtained from the first by use of Eq. (8). Since ω space is a Euclidean space, the metric coefficients g_{ij} are defined as follows[7]:

$$g_{ij}=\begin{cases}1 & \text{if } i=j\\ 0 & \text{if } i\neq j.\end{cases} \qquad (11)$$

Thus Eq. (10) may be expanded into

$$M=[(\Omega^0 a^1-\Omega^1)^2+(\Omega^0 a^2-\Omega^2)^2+(\Omega^0 a^3-\Omega^3)^2]^{\frac{1}{2}}$$
$$=\Omega^0[(a^1-\omega^1)^2+(a^2-\omega^2)^2+(a^3-\omega^3)^2]^{\frac{1}{2}}, \qquad (10a)$$

which is equivalent to equations of Section 3.

Given m colors[7] represented by $(\omega^k)_i$, where $i=1, 2, \cdots m$. The areas of the m color patches are $(\Omega^0)_i$. According to Postulates 1 and 2, the moments of the color patches should be simple multiples of one another, or

$$(M)_i=(n)_i\Omega^0[[a^1-(\omega^1)_1]^2$$
$$+[a^2-(\omega^2)_i]^2+[a^3-(\omega^3)_i]^2]^{\frac{1}{2}}.$$

The $(n)_i$ are integers 1, 2, or 3; 1 being chosen for Postulate 1 and either 2 or 3 for Postulate 2. According to either postulate,

$$(n)_1(M)_1=(n)_2(M)_2=(n)_3(M)_3$$
$$=\cdots=(n)_m(M)_m. \qquad (12)$$

Equation (12) is applicable in two particularly simple ways. First, in the case in which the colors are given and the areas are to be determined. The second case deals with the problem of coloring a given design. In the former case, the relative areas are given by the equation,

$$\frac{(\Omega^0)_i}{(\Omega^0)_j}=\frac{(n)_j[[a^1-(\omega^1)_j]^2+[(a^2-(\omega^2)_j]^2+[a^3-(\omega^3)_j]^2]^{\frac{1}{2}}}{(n)_i[[a^1-(\omega^1)_i]^2+[a^2-(\omega^2)_i]^2+[a^3-(\omega^3)_i]^2]^{\frac{1}{2}}}. \qquad (13)$$

Ordinarily, it is convenient to express all areas in terms of the largest one, say $(\Omega^0)_1$. Then the area of the ith color patch is

$$(\Omega^0)_i=\frac{(\Omega^0)_1(n)_1[[a^1-(\omega^1)_1]^2+[a^2-(\omega^2)_1]^2+[a^3-(\omega^3)_1]^2]^{\frac{1}{2}}}{(n)_i[[a^1-(\omega^1)_i]^2+[a^2-(\omega^2)_i]^2+[a^3-(\omega^3)_i]^2]^{\frac{1}{2}}}. \qquad (13a)$$

In the other special case, the areas of all patches are given. One of the colors $(\omega^k)_1$ is chosen, and the general type of harmony is decided upon. The other colors are now to be determined. Take any patch, such as the jth, whose known area is $(\Omega^0)_j$. The color to be used for this patch is not uniquely determined by Postulates 1 and 2. The designer has considerable freedom in the choice of the color but is restricted to colors which are represented by points on a sphere. The center of the sphere is at the adaptation point and the radius is

$$(R)_j=\frac{(\Omega^0)_1(n)_1}{(\Omega^0)_j(n)_j}[[a^1-(\omega^1)_1]^2$$
$$+[a^2-(\omega^2)_1]^2+[a^3-(\omega^3)_1]^2]^{\frac{1}{2}}. \qquad (14)$$

Any point on this sphere satisfies Postulates 1 and 2, and thus the designer is at liberty to satisfy also the requirements of the previous paper.[1] In this way the color for the jth patch is decided upon. Each of the $(m-2)$ other patches is treated in the same manner.

Now consider the *balance point* of a set of color patches. This concept deals with the

addition of weighted points in x space. Suppose that two color patches X and Y are given and that we wish to associate a geometric point with the sum of the two. The weighted point that is the sum of these two weighted points is

$$Z^s = X^s + Y^s. \tag{15}$$

The geometric point associated with Z^s is

$$\begin{aligned} z^k &= Z^k/Z^3 = (X^k + Y^k)/Z^3 \\ &= (X^0x^k + Y^0y^k)/(X^0 + Y^0). \end{aligned} \tag{16}$$

Similarly, the z^k for n color patches $(X^k)_i$ is

$$z^k = \sum_1^n (X^0x^k)_i / \sum_1^n (X^0)_i. \tag{17}$$

The point z^k is called the *balance point* for the set of color patches. It is the analog of the *center of gravity* of a set of mass points, and in colorspace it determines the general psychological effect of the color ensemble.

6. EXAMPLES

In a previous paper,[1] a classification was made of color harmonies, the principal division being: Class I. Harmonies in one variable (r, θ, or z in ω space); Class II. Harmonies in two variables; Class III. Harmonies in three variables. One or more examples were given for each subdivision of these harmonies.

The method of using the present paper is first to obtain a harmony without considering area. This will give a pleasing arrangement, but it can be further improved by adjusting the areas according to the method of scalar moments. An example was taken for each subdivision of color harmonies, and the correct ratio of areas was computed. The results are shown in Table IV.

The first column of the table gives the harmony classification. Spaces are provided for four colors, the Munsell specification and the relative area being given for each. In a few cases, more than four colors are used, the additional ones being placed under "Color No. 4." The final column gives the psychological effect of the harmony and is based on a computation of the balance point.

For example, under Class 1, 2, (*b*) (1) is listed the harmony *RP* 4/4, *RP* 4/8, *RP* 4/12. A calculation of moments (Table II) showed that the ratio of areas for a pleasing balance should be 1.00 : 0.79 : 0.62. The balance point for the color harmony was found to be *RP* 4/7.50; and from Table III this point gives a stimulating and slightly warm impression.

It will be noted that in a few cases *N* 5 is used as one element of a harmony. Since this element is identical with the adaptation point, it has no moment, and consequently this gray area can be of any size. All other points in colorspace are associated with definite moment arms, and the regular method of the preceding sections is applicable.

Consider now a given design for which colors are to be chosen. Figure 3 shows the outlines of an example, based on a poster designed by Professor A. Rabenbauer of Munich. The first question to be decided is the general plan of assignment of colors. It was decided to make the background (1) of one color, and to use another (2) for the face, fruit, and title block. A third color (3) is used for the basket, lips, and part of the scarf, while the remainder of the scarf is made of the fourth color (4).

The areas were measured with a planimeter and were found to be: (1) 24.3 in.^{2}, (2) 11.21, (3) 9.76, (4) 5.99. The relative moment arms are then found by dividing the largest area by each area. They are as follows: (1) 1.00, (2) 2.16, (3) 2.49, (4) 4.06. Proper area balance is obtained by comparing these relative moment arms with the numbers given in Table II.

If the design is to be limited to grays, it is practically necessary to take (1) as either *N* 6 or *N* 4, since no other gray with an integral Munsell value will allow the required range of moment arms. Accordingly, the following grays were chosen for the design: (1) *N* 6, (2) *N* 3, (3) *N* 8, (4) *N* 9. These colors are used in the achromatic harmony of Fig. 4.

A few chromatic harmonies developed in the same way are listed in Table V. Column 1 gives the area number, while column 2 lists the theoretical moment arms (relative) required for area balance. Design *A* is the achromatic harmony previously mentioned, the colors being listed with their moment arms. Designs *B*, *C*, and *D* are chromatic designs. In all cases, the actual moment arms are in approximate agreement with the theoretical.

TABLE IV. Examples of color harmony.

Classification	Color No. 1 Color	Area	No. 2 Color	Area	No. 3 Color	Area	No. 4 Color	Area	Effect of balance point
Class I *Harmonies in one variable*									
1*A* (*a*) (1)	*N* 7	1.00	*N* 8	0.67					Cheerful.
(2)	*N* 3	0.50	*N* 6	1.00					Neutral.
(*b*) (1)	*N* 6	1.00	*N* 7	0.50	*N* 8	0.33			Cheerful.
(2)	*N* 3	0.50	*N* 6	1.00	*N* 9	0.25			Neutral.
(3)	*N* 2	0.67	*N* 3	1.00	*N* 7	1.00			Neutral.
1*B* (*a*) (1)	*R* 6/8	0.71	*R* 5/8	1.00					V. stim., v. warm.
(2)	*P* 6/8	1.00	*P* 3/8	0.63					S. stim.
(*b*) (1)	*GY* 7/8	0.45	*GY* 6/8	0.71	*GY* 5/8	1.00			S. restful, neutral temp.
(2)	*R* 8/4	0.16	*R* 5/4	1.00	*R* 2/4	0.16			V. stim., v. warm.
(3)	*BG* 7/4	0.54	*BG* 6/4	1.00	*BG* 3/4	0.54			V. restful, v. cool.
2 (*a*) (1)	*YR* 4/4	1.00	*YR* 4/8	0.79					Stim., warm.
(2)	*GY* 7/2	1.00	*GY* 7/10	0.85					Cheerful.
(*b*) (1)	*RP* 4/4	1.00	*RP* 4/8	0.79	*RP* 4/12	0.62			Stim., s. warm.
(2)	*N* 4	1.00	*R* 4/7	0.75	*R* 4/14	0.50			V. stim., v. warm.
3 (*a*) (1)	*Y* 8/8	1.00	*YR* 8/8	1.00					Cheerful, stim., warm.
(2)	5 *R* 7/8	1.00	10 *GY* 7/8	1.00					Cheerful.
(*b*) (1)	*RP* 3/10	1.00	*P* 3/10	1.00	*PB* 3/10	1.00			Gloomy, s. stim., n. temp. effect.
(2)	*G* 7/2	1.00	*YR* 7/2	1.00	*P* 7/2	1.00			Cheerful.
(3)	*GY* 6/6	1.00	*G* 6/6	1.00	*P* 6/6	1.00			Neutral.
Class II *Harmonies in two variables*									
1*A* (*a*) (1)	*BG* 5/6	1.00	*BG* 4/4	0.67					V. restful, v. cool.
	GY 6/6	0.80	*GY* 5/8	1.00					S. restful, n. temp. effect.
(2)	*YR* 5/6	1.00	*YR* 4/2	0.73					Stim., warm.
	GY 7/4	0.61	*GY* 5/10	1.00					S. restful, n. temp. effect.
(*b*), (1)	*G* 5/2	1.00	*G* 6/4	0.22	*G* 7/6	0.12			Neutral.
	P 6/8	0.88	*P* 5/10	1.00	*P* 4/12	0.69			S. stim., n. temp. effect.
(2)	*R* 7/6	0.52	*R* 4/4	1.00	*R* 1/2	0.28			V. stim., v. warm.
	RP 8/2	0.25	*RP* 5/6	1.00	*RP* 2/10	0.23			Stim., s. warm.
(3)	*GY* 5/2	1.00	*GY* 7/8	0.11	*GY* 8/10	0.08			Neutral.
	PB 7/4	0.61	*PB* 6/6	1.00	*PB* 4/10	0.78			Non-stim., cold.
1*B*, (*a*), (1)	*R* 5/6	1.00	*R* 4/4	0.67	*R* 4/8	0.53			V. stim., v. warm.
(2)	*PB* 7/6	0.48	*PB* 4/2	1.00	*PB* 4/10	0.64			V. stim., n. temp. effect.
(3)	*YR* 7/4	0.50	*YR* 4/2	1.00	*YR* 4/6	0.83			Stim., warm.
(*b*), (1)	*N* 5.5	0.50	*G* 6/2	0.24	*G* 5/2	1.00			Neutral.
(2)	*RP* 5/2	1.00	*RP* 2/4	0.08	*RP* 8/4	0.08			Neutral.
(3)	*N* 4.5	1.00	*BG* 5/8	0.50	*BG* 4/8	0.35			V. restful, v. cool.
(*c*), (1)	*BG* 6/6	0.40	*BG* 5/4	1.00	*BG* 5/6	0.67			V. restful, v. cool.
(2)	*Y* 8/4	0.16	*Y* 8/12	0.15	*Y* 5/4	1.00			Cheerful, s. stim., s. warm.
(3)	*B* 7/2	1.00	*B* 7/6	0.94	*B* 3/6	0.94			Non-stim., cool.
1C, (*a*)	*YR* 7/6	0.59	*YR* 7/10	0.53	*YR* 6/6	1.00	*YR* 6/10	0.78	Cheerful, stim., warm.
(*b*)	*N* 6	1.00	*B* 6/8	0.80	*N* 3	0.50	*B* 3/8	0.45	Neutral.
(*c*)	*B* 7/2	0.51	*B* 7/6	0.48	*B* 4/2	1.00	*B* 4/6	0.83	Non-stim., cool.
1*D*	*R* 6/6	0.83	*R* 4/2	1.00	*R* 4/10	0.65	*R* 2/6	0.33	V. stim., v. cool.
2, (*a*), (1)	*RP* 7/4	1.00	*R* 7/8	0.92					Cheerful, s. stim., warm.
(*b*), (1)	*N* 7	1.00	*BG* 7/4	0.97	*G* 7/4	0.97			Cheerful.
(*c*)	*G* 5/4	1.00	*BG* 5/4	1.00	*N* 5	—	*RP* 5/4	1.00	Neutral.
							R 5/4	1.00	
(*d*)	*YR* 6/6	1.00	*RP* 6/6	1.00	*PB* 6/6	1.00	*BG* 6/6	1.00	Neutral.
							GY 6/6	1.00	
(*e*)	*R* 5/6	0.33	*P* 5/4	0.50	*BG* 5/2	1.00	*Y* 5/4	0.50	Neutral.
3, (*a*), (1)	*GY* 7/6	0.58	*G* 6/6	1.00					Cheerful, restful, s. cool.
(2)	*G* 3/2	1.00	*P* 7/2	1.00					Neutral.
(3)	*R* 7/8	1.00	*RP* 3/8	1.00					Stim., v. warm.
(4)	*YR* 7/4	0.54	*BG* 6/4	1.00					Cheerful.
(*b*), (1)	*R* 2/4	0.68	*G* 7/4	1.00	*B* 7/4	1.00			Neutral.
(2)	*GY* 8/6	0.40	*R* 4/6	1.00	*PB* 4/6	1.00	*P* 2/6	0.40	Neutral.

TABLE IV.—(*Continued*).

Classification	Color No. 1 Color	Area	No. 2 Color	Area	No. 3 Color	Area	No. 4 Color	Area	Effect of balance point
Class III *Harmonies in three variables*									
1, (*a*)	*P* 6/8	1.00	*P* 3/8	0.63	*RP* 6/8	1.00			S. stim., n. temp. effect.
(*b*)	*R* 8/4	0.16	*R* 5/4	1.00	*R* 2/4	0.16	*BG* 5/4	1.00	Neutral.
(*c*)	*P* 6/8	1.00	*P* 3/8	0.63	*B* 6/8	1.00	*B* 3/8	0.63	N. stim., cold.
(*d*)	*N* 4	1.00	*R* 7/6	0.47	*R* 6/4	0.90	*B* 7/6	0.47	Neutral.
							G 7/6	0.23	
2, (*a*)	*N* 6	0.50	*R* 6/4	0.45	*BG* 6/4	0.45	*BG* 5/4	1.00	Neutral.
(*b*)	*RP* 7/4	1.00	*R* 7/8	0.92	*R* 3/8	0.92			Stim., warm.
(*c*)	*BG* 6/2	1.00	*R* 6/10	0.64	*BG* 2/2	0.34	*R* 2/10	0.32	Neutral.
(*d*)	*RP* 3/2	0.51	*R* 3/10	0.44	*RP* 7/2	0.51	*RP* 6/2	1.00	Neutral.
3, (*a*)	*GY* 8/6	0.40	*R* 4/6	1.00	*PB* 4/6	1.00	*P* 2/6	0.40	Neutral.
							R 4/10	0.78	
(*b*)	*N* 5.5	1.00	*B* 6/4	0.45	*R* 5/4	1.00	*R* 5/12	0.33	Neutral.
(*c*)	*G* 3/2	1.00	*P* 7/2	1.00	*G* 3/6	0.94	*P* 7/6	0.94	Neutral.
(*d*)	*R* 7/8	0.90	*RP* 3/8	0.90	*R* 7/4	0.97	*N* 7	1.00	Stim., warm.
4, (*a*)	*N* 5	—	*GY* 2/2	1.00	*PB* 8/2	1.00			Neutral.
(*b*)	*P* 3/10	0.60	*PB* 4/8	1.00	*RP* 4/8	1.00	*GY* 7/4	0.68	Neutral.

TABLE V. Color harmonies for Fig. 3.

Area No.	Theoretical moment arms	Design *A*		Design *B*		Design *C*		Design *D*		Design *E* Theo.	Design *E* Actual	
1	1.00	*N* 6	1.0	*R* 6/2	1.0	*BG* 6/2	1.0	*BG* 5/4	1.0	1.00	*R* 5/4	1.0
2	2.16	*N* 3	2.0	*R* 3/2	2.0	*YR* 3/2	2.0	*R* 4/2	2.1	2.16	*R* 4/2	2.1
3	2.49	*N* 8	3.0	*R* 3/10	2.3	*R* 3/10	2.3	*R* 4/6	2.5	5.65	*R* 3/10	4.7
4	4.06	*N* 9	4.0	*N* 9	3.9	*N* 9	3.9	*BG* 7/4	4.1	4.45	*R* 7/6	4.3
5										4.06	*G* 7/2	4.0

The approximation is probably good enough for all practical purposes, though even better agreement could have been obtained by using colors not in the Munsell list.

The final design (*E*) of Table V is a 5-color arrangement in which the scarf is made in two colors which are not used elsewhere.

7. MUNSELL'S RULE

After the completion of the work described in this paper, we found a quantitative rule for areas in the writings of A. H. Munsell. He says,[9] "The general law being that the stronger the color . . . the smaller must be its area; while the larger the area, the grayer the Chroma. Thus *R* 7/6 balances *R* 3/3 in the proportion of nine parts of the lighter red to forty-two parts of the darker red. In other words, these symbols will balance colors *inversely* as the product of their factors. This opens up a great field of *area*."

The same rule is stated by T. M. Cleland in the words,[10] "We now have to take the Value into account in determining the amount of area of each of these two colors to be used if we are to arrive at a perfectly balanced color design; and this is done by the simple process of multiplying the Chroma by the Value of each of the colors."

It is interesting to note that Munsell's rule is identical with ours for the special case where all colors are of value 5. Then the moment arms are equal to the chromas, and consequently the areas required for balance are inversely proportional to the chromas.

For other conditions, however, the two methods give widely divergent results. For the colors (*R* 7/6, *R* 3/3) mentioned by Munsell in the foregoing quotation, he gives an area ratio of 1 : 4.7. But according to Table II, the moment

[9] A. H. Munsell, "An introduction to the Munsell color system," in *A Grammar of Color* (Strathmore Paper Company, 1921), p. 9.

[10] T. M. Cleland, "A practical description of the Munsell color system," in *A Grammar of Color* (Strathmore Paper Company, 1921), p. 22.

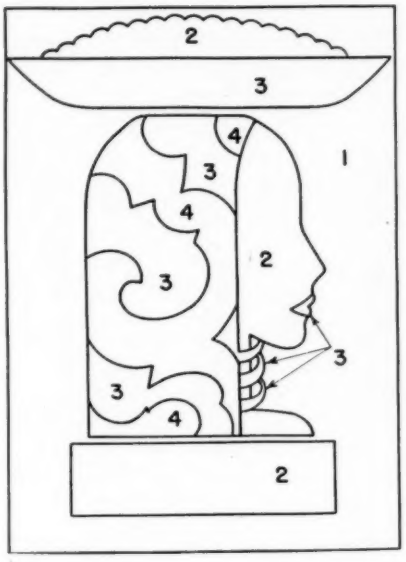

FIG. 3. Outline of a poster design.

arms are 17.1 and 16.3, resulting in an area ratio of 1.00 : 1.05. Geometrically, the product of value and chroma gives the area of a rectangle in a plane $\theta = \text{const}$. One corner of the rectangle is at the colorpoint P and the opposite corner is at the origin of coordinates (Fig. 1). Thus, according to Munsell's rule, the area of the color patch must be inversely proportional to the area of this rectangle in ω space. There seems to be no justification for such a rule, which obviously breaks down completely if one of the points is on the neutral axis (chroma = 0).

Munsell's idea can be tested most easily by use of Munsell color chips. Two chips, $R\,3/10$ and $R\,5/6$ (or RP or P), are placed on a large background of $N\,5$. The products are 30 in each case; so according to Munsell's rule, equal areas will give a sensation of balance. Actually, the $R\,5/6$ appears weak, and a feeling of equality and balance is only found when the area of $R\,3/10$ is reduced to approximately 1/3 in accordance with the theoretical moment arms 18.9 and 6 (Table II). For these reasons the Munsell rule for area balance appears to be invalid.

FIG. 4. Gray harmony applied to the design of Fig. 3. The colors are (1) $N\,6$, (2) $N\,3$, (3) $N\,8$, (4) $N\,9$.

8. CONCLUSIONS

The paper employs the two concepts: (1) *scalar moment* of a color patch about the adaptation point in ω space, and (2) the *balance point* of a set of colors. These concepts are used in extending the scientific formulation of color harmony to include the *areas* of the color patches. It is found that a pleasing sense of balance is obtained when all the color patches have equal scalar moments about the adaptation point. Other pleasing arrangements occur when the moments are simple multiples of each other. The second concept—that of the balance point—is useful in determining the general psychological effect of a harmony. The whole subject makes an interesting application of the mathematical concept of weighted points.

In a practical application of scientific color harmony, the designer first selects hues that he considers to be appropriate to the application. He then decides on the type of harmony to be used and selects either large or small color steps,

avoiding the regions of ambiguity.[1] Areas are then determined according to the methods of the present paper. Ordinarily the areas are adjusted so that the scalar moments of the various color patches are equal; but if one of the colors is to be emphasized, its area can be increased to give either 2 or 3 times the moment of the other color patches. Finally, the designer calculates the balance point of his color ensemble and decides if the psychological effect is suitable or if he should modify his colors. All this he does by simple mathematics, without the use of any colorants whatever; just as the musician harmonizes a melody by the application of well-known rules, without touching the keyboard.

Another problem occurs when the outline of the design has been fixed—that is, the areas are given—and satisfactory colors are to be chosen. This case is solved by the application of the same principles. In either case, it is evident that the new method of color harmony does not stultify artistic ability. On the contrary, it suggests many new types of harmony and it reduces the time otherwise wasted in groping experimentally for a satisfactory color arrangement.

Spectroscopic Analysis of Magnesium Alloys*

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IN 1935 this laboratory reported on the spectrographic analysis of magnesium alloys for manganese and silicon.[1] Since that time we have continued to use this method of analysis in controlling the composition of our alloys, expanding its use to cover the analysis for twenty-five different elements in magnesium and its alloys as the need for such analysis has arisen. At the present time the Midland laboratory is making approximately 2000 quantitative determinations on magnesium each twenty-four-hour period.

The basis of the technique is the experimental determination of the relationship between the intensities of selected lines of a constituent of a specimen and of an internal standard element[2] in the specimen which in this case is the major constituent, magnesium.

ROUTINE OPERATING CONDITIONS

Eastman Spectrum Analysis No. 1 plates are used for all alloy analyses. They are developed 2½ minutes in D-19 at 67°F, hardened for 30 seconds in chrome alum hardener, and fixed one minute in an acid hardening fixing bath. After washing one minute the plate is rinsed in distilled water and dried.

An iron spark spectrum is photographed on each plate in addition to the alloy spectra. This iron spectrum is used to determine the photographic response of the plate. For a period response curves were drawn only once or twice each day and whenever emulsion numbers were changed. We have found, however, with inexperienced operators that much better results are obtained if the response curve is drawn for each plate.

The exposure and timing mechanism is so arranged that the operator has ample opportunity to prepare the next pair of electrodes for analysis while the first sample is being photographed. The timing mechanism is described in another paper[3] from this laboratory.

Part of our analytical load consists of inspection analysis and another part of control analysis. The operating schedule for the inspection analysis is given in Table I. The usual speed for handling the control analysis is 20 minutes' total elapsed time per sample, analyzed for an average of 8 elements.

The lines used for the analysis of each element with their reference lines are given in Table II.

Table I. Operating schedule.

Photographing plate containing nine samples and iron calibration		
Prespark	15 sec.	
Spark exposure	40 sec.	
Pre-arc	5 sec.	
Arc	20 sec.	
Total	80 sec. × 9 samples	12 minutes
Iron calibration		½ min.
Loading electrodes and making out necessary legend		8½ min.
Processing and loading plate		10 min.
Reading, calculating and recording plate		45 min.
Total man minutes required to analyze 9 samples for 9 elements each		76 minutes

SOURCES

Two sources of excitation are used depending upon the composition range of the element sought. In analyzing for such elements as aluminum, calcium, copper, manganese, silicon, and zinc the conventional controlled spark[4,5] discharge is used. The constants of the circuit are so selected that the magnesium lines 3074A and 2736A are approximately of equal intensity. Although a circuit with less inductance is better for some determinations such as the calcium determination, this circuit has been found to be

* Paper presented at the Twenty-Eighth Annual Meeting of the Optical Society of America, Pittsburgh, Pennsylvania, October 7–9, 1943.

[1] "Quantitative spectrographic analysis of magnesium alloys for manganese and silicon," Proc. A.S.T.M. **35**, Part II, 61–70 (1935).

[2] Walther Gerlach and Eugen Schweitzer, *Die Chemische Emissions-Spektral analyse, Grundlagen und Methoden* (Leopold Voss, Leipzig, 1930).

[3] J. L. Saunderson and V. J. Caldecourt, J. Opt. Soc. Am. **34**, 116 (1944).

[4] O. Feussner, Arch. Eisenhüttenwesen **6**, 551 (1932–1933).

[5] H. B. Vincent and R. A. Sawyer, J. App. Phys. **8**, 163 (1937).

TABLE II. Line pairs used in analysis.

Element	Percentage range	Arc Reference line	Unknown line	Element	Percentage range	Spark Reference line	Unknown line
Cd	0.001–0.1	2698 Mg or B.G.	2288	Al	0.001–0.1	4167 Mg	3962 Al
Fe	0.001–0.05	2698 Mg or B.G.	2483		0.05–2.0	3074 Mg	3082 Al
Fe	0.001–0.05	2698 Mg or B.G.	2488		0.4–12.0	2736 Mg	2568 Al
Ni	0.001–0.05	2698 Mg or B.G.	3415	Ca	0.01–0.50	3074 Mg	3179 Ca
Pb	0.001–0.15	2698 Mg or B.G.	2614	Cd	0.1–6.0	3074 Mg	3261 Cd
Pb	0.01–0.5	2698 Mg or B.G.	2663	Cu	0.001–0.3	3074 Mg	3274 Cu
Sn	0.001–0.3	2698 Mg or B.G.	3175	Mn	0.001–0.01	2736 Mg	2606 Mn
				Mn	0.01–0.5	3074 Mg	2950 Mn
				Mn	0.3–3.0	2736 Mg	2610 Mn
				Pb	0.1–6.0	2736 Mg	2614 Pb
				Si	0.01–1.5	2736 Mg	2516 Si
				Sn	0.1–6.0	3074 Mg	3034 Sn
				Zn	0.01–1.0	3074 Mg	3303 Zn
					0.3–4.0	3074 Mg	3282 Zn

B.G. = background.

the best compromise for all the determinations ordinarily made. The discharge conditions are:

1. Transformer—of sufficient size to charge the condenser to 25,000 volts at the moment of discharge.
2. Parallel capacitance—0.0053 μf.
3. Series inductance—950 microhenrys.
4. Synchronous interrupter—tuned for maximum secondary current. Average secondary current—3.6 amp.
5. Prespark—15 sec., exposure 40 sec.
6. Electrode separation—3.5 mm.

The above circuit or one containing 3000 microhenrys added inductance can also be used in analyzing for such trace impurities as iron, nickel, lead, tin, and cadmium. These spark discharges are from two to five times less sensitive than either the 2200-volt, 2.5-amp. a.c. or the 220-volt, 2.5-amp. d.c. arcs as we use them. In the analysis of high purity alloys this difference in sensitivity is very important. The arc discharge is run from the same electrodes used for the spark, immediately after the spark exposure. The mechanism for doing this automatically is also described in the paper mentioned above.[3]

There is very little difference between the a.c. and d.c. arcs as far as the analysis of magnesium alloys is concerned. In fact, if all other conditions are maintained the same, identical working curves can be used in analyzing for iron and nickel by either source.

ELECTRODES

Solid alloy electrodes cast in the form of 5-mm diameter pencils are used as test specimens. The discharge end of each electrode is machined by means of a lathe tool to a diameter of 4 mm and the discharge surface is cut perpendicularly to the axis of the rod. Electrodes with discharge surfaces beveled from 5° to 20° have been tried, but the flat-end electrodes have been found to give the most consistent results. Casting the rods 5 mm in diameter and turning down the discharge end to 4 mm has two definite advantages. First, the larger mass of metal contained in a 5-mm rod as compared to a 4-mm rod dissipates the heat from the arc more readily, thus producing a more steady discharge. Second, the chance of contamination from the mold wall is eliminated by removing the outside layer of metal. By using a discharge surface 4 or 5 mm in diameter, the surface is uniformly worked during the exposure period, whereas a much larger surface would not be uniformly worked.

SAMPLING

In any analytical method the selection and physical state of the sample are very important. This is particularly true in the case of spectrographic analysis where a very minute sample must often be representative of a large mass. The sample should be fine grained and free from microporosity. Table III shows microchemical

TABLE III. Chemical and spectrographic determination of the zinc content of porous and non-porous areas of magnesium casting.

Spec.	% Zn (Spect.)	% Zn (Chem.)
H8259S	3.2	3.02
P	2.6	2.80
30696–9*S*	2.8	2.91
P	2.45	2.50
30696–13*S*	2.9	2.80
P	2.65	2.64

TABLE IV. Composition of porous and non-porous areas.

Spec.	% Al	% Mn	% Zn
H8550S	6.2	0.13	3.0
H8550P	5.8	.12	2.6
H8558S	6.1	.25	3.1
H8558P	5.8	.25	2.6

and spectrographic analyses for Zn on a porous area and non-porous area in the chill cast specimens; *P* is the porous area as located by the radiograph and *S* is an area of solid metal.

The aluminum content of these samples will vary in the same manner as the zinc and to the same degree. The reason that these areas of microporosity are lower in aluminum and zinc than the rest of the specimen is that the first metal to freeze in this type of alloy contains less aluminum and zinc than the lower melting portion. If this area is trapped off as the metal freezes so that the molten fraction cannot flow back into these microvoids, the resulting metal will be lower in aluminum and zinc than the average and the adjoining area higher than the average.

Although the aluminum and zinc segregate in this manner, there is little or no evidence of manganese segregation in these same areas. Table IV shows two samples which illustrate the segregation of aluminum and zinc and yet the manganese shows no tendency to segregate.

It is necessary, therefore, to cast the test specimen in such a manner that porosity is avoided while the fine grain structure of a chill casting is maintained. A type of casting which we have found very satisfactory for test specimens is illustrated in Fig. 1. The castings are made in graphite or iron-split molds. The mold is heated to a temperature of about 300°F before pouring the sample. The metal is poured at temperatures varying from 1200°F to 1300°F depending upon the type of alloy being sampled. The test specimen is the section 5 mm in diameter and 75 mm long at the bottom of the casting which is cut from the rest of the casting on a small band saw. This rod is then cut in the middle and the two electrodes thus formed are machined as described above.

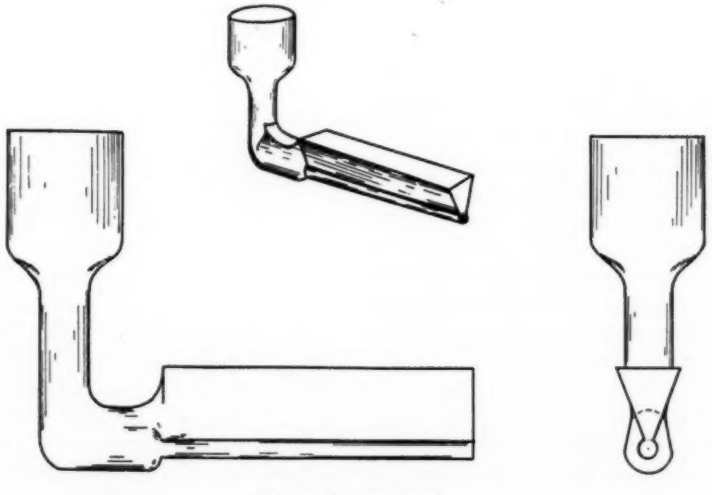


FIG. 1. Test specimen.

In analyzing for manganese and silicon in the magnesium, aluminum, zinc alloys, another type of problem may be encountered. Figure 2 illustrates the change in the ratio of the intensity of the magnesium to silicon, caused by increasing the size of the silicon particles or grains. The lower curve was obtained from the analysis of chill cast samples. The upper curves represent slowly cooled samples of the same melts containing the same concentration of silicon. The rate of cooling was controlled by casting the series of samples represented by the top curve in 1600-gram castings, the series represented by the middle curve in 150-gram castings, and the third series in the form of chill cast bars. That these three samples from each melt do actually contain the same silicon concentration was confirmed by chemical analysis. The main difference in the three samples is in grain size caused by different cooling rates. This effect is most pronounced in the magnesium, aluminum, zinc type of alloy and is not evident in the Mg-Si alloy. As the aluminum or zinc concentration increases, this effect becomes more pronounced. Table V illustrates a similar effect in the case of manganese. Specimens 9715-1A, -1B, -1C, -1D are samples cut from twenty-pound notched ingot. Specimen 9715-T-1 and T-2 are two test castings made from a small melt containing only ingot 9715-1. 9715 is the original test specimen cast from the larger melt from which ingot 9715-1 was poured. It will be noted that the chemical value for manganese and the spectrographic value are in good agreement on the chill cast specimens 9715-T-1 and 9715-T-2. The spectrographic values obtained on the samples cut from the ingot are low in every case. Figure 2 shows the difference in the grain size of these two specimens cast from the same metal. Since the analytical curves are based on chill cast samples, the analysis on the chill cast electrodes should agree with the chemical analysis. The analysis of the other samples is low because the larger manganese particles are not volatilized into the discharge column as rapidly as the small uniformly dispersed particles of the chill cast specimen. The same explanation accounts for the differences in silicon analysis on the samples of different grain size mentioned above.

The obvious way to avoid this error is to base the analytical curves on standards made under

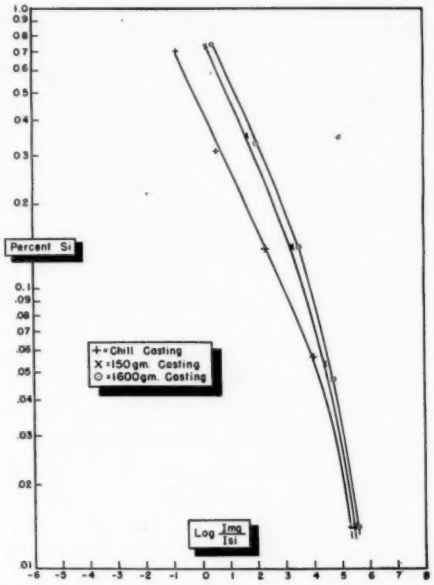


FIG. 2. Change in the ratio of the intensity of magnesium to intensity of silicon caused by a difference in the size of the silicon grains.

conditions similar to the conditions to be used in making the test sample. There is a possibility that a different form of excitation might reduce or eliminate this effect. Our efforts in this direction have so far been unsuccessful.

Frequently the spectrographic laboratory or chemical laboratory is asked to analyze ingots of metal taken at random from shipments of metal. In the case of magnesium alloys these ingots usually contain twenty pounds of metal. The analyst must decide how to sample this large mass of metal in order to get an analysis representative of the whole. Table V illustrates the

TABLE V. Spectrographic and chemical analysis of various samples of the same metal cast in different ways.

Spec.	Description	% Al		% Mn		% Zn	
		Spect.	Chem.	Spect.	Chem.	Spect.	Chem.
9715	Chill cast test spec.	5.9	6.1	0.29	0.30	3.0	2.96
9715-1-A	Ingot section	6.5	6.23	.20	.32	3.15	3.13
-1-B	Ingot section	5.7	5.86	.24	.31	2.9	2.41
-1-C	Ingot section	6.3	6.17	.23	.33	3.17	3.15
-1-D	Ingot section	5.3	5.17	.29	.33	2.5	2.35
-T-1	Chill cast test spec.	6.2	6.13	.30	.31	3.0	3.04
-T-2	Chill cast test spec.	6.0	6.03	.31	.31	3.0	3.13

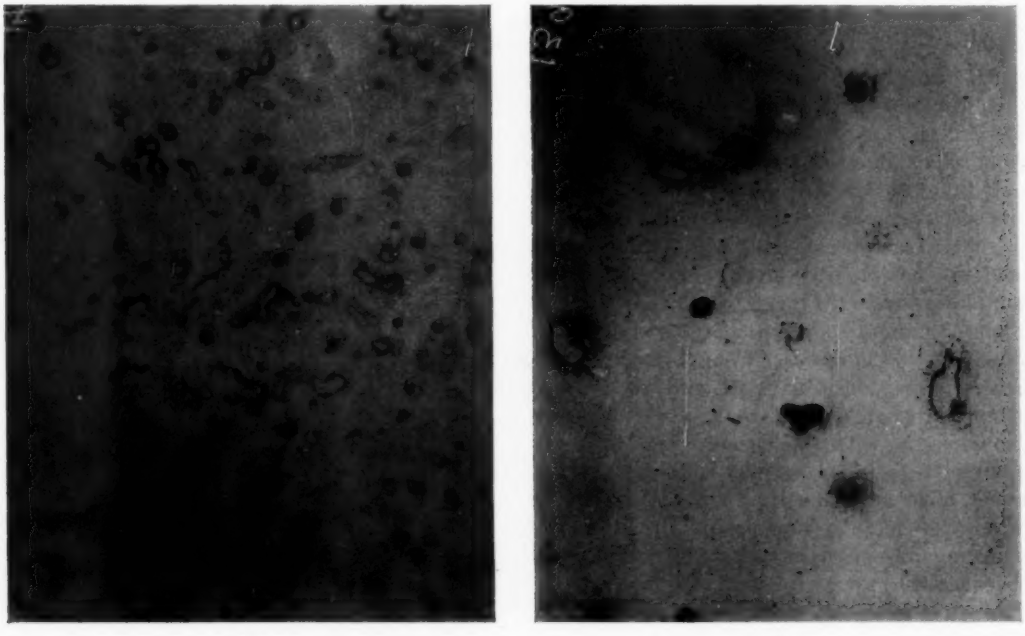
TABLE VI. Typical precision data.

	% Al	% Cu	% Mn	% Zn
Composition	6.3	0.023	0.22	3.36
Standard deviation	±.15	±.0013	±.01	±.12

results one may expect to obtain if he samples a perfectly satisfactory ingot of magnesium of the 3 percent Zn, 6 percent Al, 0.2 percent Mn type, at the points indicated in Fig. 3. Although the chemical manganese determination shows that the manganese is quite uniformly distributed throughout the entire ingot, the aluminum and zinc will be badly segregated. The type of segregation encountered in this case is known as "inverse" segregation. A complete discussion of this subject is a metallurgical problem and beyond the scope of this paper. It may be explained briefly as follows. As the metal solidifies, it contracts, causing a very high pressure to be exerted on the molten metal at the center of the casting. This pressure forces the molten metal, which will contain more aluminum and zinc than the average, through the interstitial channels toward the surface. A sample taken from the area *A*, Fig. 4, will be most nearly representative of the metal as a whole in this type of casting. The least representative sample will be one taken from areas *B* or *D*. The point to be stressed is the

TABLE VII. Comparison of chemical and spectrographic determinations for aluminum manganese and zinc.

Alloy	Chemical % Al	Spect. % Al	Chemical % Mn	Spect. % Mn	Chemical % Zn	Spect. % Zn
*H*785	5.6	5.6	0.229	0.22	3.0	2.9
786	5.7	5.8	.219	.22	3.0	2.9
787	5.6	5.6	.226	.21	3.0	2.8
788	5.7	5.6	.413	.40	2.9	3.1
789	5.9	6.2	.224	.21	3.1	3.2
790	6.2	6.2	.248	.25	3.1	3.2
791	6.2	6.0	.26	.25	3.0	2.9
792	6.5	6.5	.25	.26	3.2	3.1
793	6.0	5.8	.224	.21	3.0	3.2
794	6.0	5.8	.224	.21	3.2	3.3
795	6.1	5.8	.15	.17	3.1	3.1
796	5.7	5.8	.18	.17	2.6	2.6
797	5.9	5.8	.17	.17	2.6	2.4
798	5.8	6.0	.16	.16	3.4	3.4
799	6.1	6.1	.15	.15	3.5	3.5
800	6.2	6.1	.13	.14	3.4	3.5



(a) (b)

FIG. 3. (a) Grain size of chill casting 9715-T-1 (500×). (b) Grain size of section from ingot 9715-1-A (500×). The dark areas with the light center are the manganese particles.

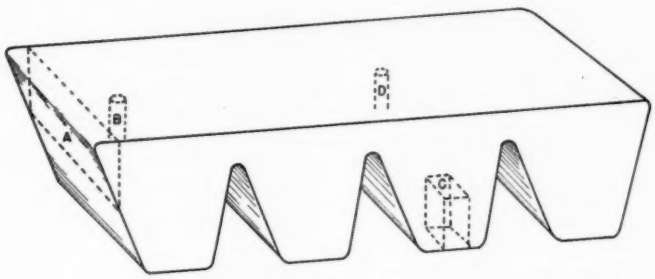


FIG. 4. Ingot of magnesium alloy sampled at points indicated.

importance of exercising extreme care in sampling any large casting. Whenever possible it is advisable to melt the entire ingot and cast a test specimen from the melt.

PRECISION

When all the factors mentioned above are controlled the precision obtained with the method described is represented by Table VI. The data for Table VI were obtained by analyzing samples from the same melt in the following manner: One sample was recorded and analyzed on each routine plate for 95 consecutive plates. The standard deviation was calculated in the usual manner from these analyses.

TABLE VIII. Comparison of chemical and spectrographic determinations for iron.

Sample	% Fe Spectrographic A	B	Chemical C	D
5581	<0.0004	0.0009	0.0002	
5586	< .0004	.0009	.0004	
6468	.002	.0014	.0017	
6730	< .0004	.0007	.0002	
6776	.0015	.0012	.0016	0.0021
6888	.0010	.0010	.0010	.0014
6896	.0008	.0003	.0006	
6897	.0005	.0004	.0006	
1199	.013	.011	.013	
1204	.003	.0035	.0038	
1234	.005	.0062	.0067	
720	.004	.003	.0038	
721	.003	.004	.0038	

A—Original spectrographic value used to certify batch.
B—Check analysis run immediately before sending the electrodes to the chemistry laboratories for analysis.
C—Determined by the α,α' dipyridyl colorimetric method, with the aid of a photoelectric colorimeter.
D—Determined by HBr method, similar to that in A.S.T.M. methods, but with the use of a photoelectric colorimeter and correction for effect of copper.

In Table VII is to be found a comparison of the chemical and spectrographic analyses for aluminum, manganese, and zinc.

Since small amounts of iron and nickel have a detrimental effect on the corrosion resistance of magnesium alloys, their determination becomes a matter of importance especially in alloys of high purity. The arc discharge gives the necessary sensitivity but also produces a variable background. When the magnesium line (2698A) is used as an internal standard for this determination, the background variation is the largest single source of error. By using the background next to the iron line as a reference instead of a magnesium line, the accuracy given in Table VIII is regularly obtained. This accuracy is on the average twice as good at concentrations of 0.001 percent to 0.005 percent as the accuracy obtained by using the magnesium 2698A line as internal standard. Above 0.01 percent iron the accuracy of the two procedures is about equal. The alloys used for the comparison in Table VIII were picked at random from our previous months' control samples of high purity alloys. The selected samples were analyzed a second time spectrographically and the electrodes then were sent to the chemistry laboratory for analysis.

CONCLUSION

This paper has mentioned specifically only the ten elements commonly found in magnesium alloys either as impurities or alloying constituents. However, the procedure described has been found to yield satisfactory analysis for any of the metals which can be added to magnesium.

A New Optical Bench*

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APPLIED optics in the industrial field usually involves two-dimensional or three-dimensional systems rather than systems whose axes form a single straight line. In these cases the standard linear optical bench finds a very limited application. Also, precision work on the straight-line type of bench is difficult because with standard laboratory equipment the usual translational adjustment available for a given bench clamp position is only a vertical one. Hence, in order to get three-dimensional adjustments and to make two-dimensional or three-dimensional set-ups, it is often necessary to use a maze of clamps mounted on the bench riders as well as additional clamps to hold external stands in their proper position.

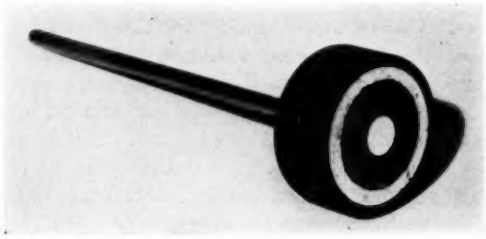

FIG. 1. Alnico magnet stand.

A new optical bench which obviates these difficulties consists of a steel base plate upon which are mounted powerful Alnico magnets into which are inserted vertical rods. A stand of this type is shown in Fig. 1. These stands can be rotated about a vertical axis or easily slid to any desired position. Whereas it often required several minutes to adjust various lenses or light sources by the old method, it is but a matter of seconds to make the necessary adjustments with the new.

Fiber paper or cardboard of the correct thickness cemented to the base of the Alnico stands will adjust the air gap between the stands and the base plate so that the desired force of attraction is obtained between the stand and the plate. This separating disk also permits the stands to

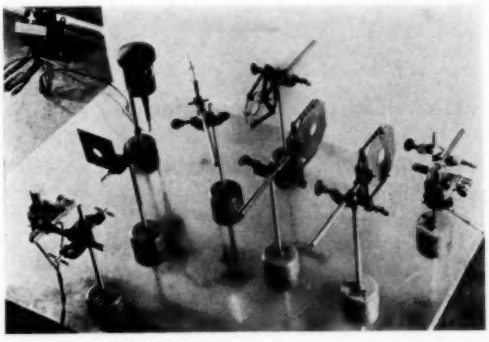

FIG. 2. The new optical bench with Alnico magnet stands.

be more easily slid along the base plate. A typical optical system is shown as used with the Alnico stands in Fig. 2.

Since these magnetic stands can be used wherever a steel base is available, they are particularly useful where a vertical system is required, since they will clamp themselves firmly to steel panels or uprights.

Alnico is an alloy of aluminum, nickel, cobalt, and iron. This alloy can either be cast or made in sintered form, the latter being produced by subjecting a mixture of the constituent metals to very high pressure and then to high temperature. The maximum external energy of this alloy when magnetized is higher than any other permanent-magnet material, and it occurs at a

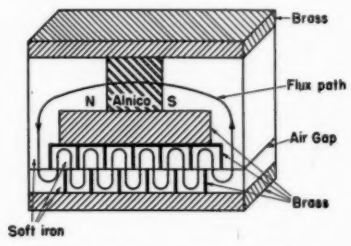

FIG. 3. Alnico magnet assembly.

* Paper presented at the Twenty-Eighth Annual Meeting of the Optical Society of America, Pittsburgh, Pennsylvania, October 7–9, 1943.

lower flux density. Hence, Alnico magnets can be of a smaller volume than other permanent magnets.

Special magnet assemblies will allow Alnico magnets to lift as much as 4450 times their own weight. Such an assembly has a segmented armature and a segmented shunt such that the magnetic circuit carries the flux across the air gap a number of times in series rather than twice, as is usually the case, thereby producing a more effective utilization of the flux. A schematic diagram of such an assembly is shown in Fig. 3.

The use of Alnico in magnetic stands should not only find wide application in industry, but also in the academic field where numerous laboratory set-ups are continually being made.

Establishment of Munsell Color Foundation

THE science of color in this country covers an active and growing field. A recent event of real importance is the establishment of the Munsell Color Foundation, Inc.

The Munsell Color Company for twenty-two years has carried on the task passed on by Mr. Albert H. Munsell at the close of his lifetime of color research and teaching. His family, as owners of the Company, have seen the theories and practical suggestions originated by Mr. Munsell carried into many fields, and have been pleased to know that his system of color notation is recognized by an increasing number of artists, scientists, industrialists, educators, and the general public.

Believing that the time had arrived when the ownership of the Company and the responsibility for carrying on this work should be placed in the hands of the persons having the broadest knowledge of color and its problems, and being most interested in the advancement of color knowledge, the owners have taken part in the creation of a foundation whose corporate name is "The Munsell Color Foundation, Inc.," and have transferred to this corporation their stock in the Munsell Color Company, Inc.

The purpose of the Foundation is to:

> Further the scientific advancement of color knowledge, and in particular, knowledge relating to the standardization, nomenclature, and specification of color; and to promote the practical application of these results to the color problems arising in science, art, and industry.

The Foundation is a non-profit organization whose management is intrusted to a Board of Trustees selected as follows:

> One . . . shall be a member of the staff of the National Bureau of Standards who shall have been designated by the Director of said Bureau; one . . . shall be designated by the Executive Committee of the Inter-Society Color Council; one . . . shall be the person who, from time to time, shall be the manager of the Munsell Color Company, Inc.; one . . . shall be the representative of Juliet E. Orr Munsell and her son, Alexander E. Orr Munsell; . . . there shall be three Trustees at Large.

The contribution made to color science by the Optical Society of America was recognized by the three special trustees of the Foundation who requested that a representative of the Society be named as one of the trustees at large. Our Board of Directors complied with this request, naming for this honor and responsibility Dr. Loyd A. Jones, Chairman of our Committee on Colorimetry.

The first Board of Trustees consists of Deane B. Judd, trustee appointed by the Director of the National Bureau of Standards, elected President; Loyd A. Jones, trustee at large nominated by the Optical Society of America, elected Vice President; Blanche R. Bellamy, Manager of the Munsell Color Company, Inc., elected Secretary; Dorothy Nickerson, trustee appointed by the Inter-Society Color Council; Alexander E. O. Munsell, trustee representing the Donor; and Arthur S. Allen and I. H. Godlove, trustees at large. Each member of this board is already well known in color science or in industry.

The Munsell Foundation will be financed by such profits as are made by the Munsell Color Company, Inc. and, it is hoped, by the help of its friends. While the thought back of this Foundation has been the extension of the Munsell Color System, care has been used to avoid including the name of the Munsell Color System in the declared purpose of the Foundation. It is felt that the system has demonstrated its utility in the past and that it can be kept abreast of modern developments by suitable improvements so as to be useful for a long time in the future; but no attempt has been made to interfere with the trustees in the free exercise of their judgment.

It is to be expected that the Munsell Color Company will, under the guidance of the Foundation, continue the services it has heretofore rendered, and will when possible extend its activities into new fields as its work is needed.

Spectrophotometric Formulation*

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(Received November 10, 1943)

Methods are presented whereby the formulation of colorants needed to effect a visual color match of a given standard may be computed from spectrophotometric measurements of the colorants and standard.

METHODS for predicting from spectrophotometric data the visual equality or lack of equality of color of two samples having spectrophotometric curves of different shapes by methods of tristimulus integration are well known.[1,2] Also several methods have been developed for predicting the spectrophotometric curve of a mixture from the curve of the individual components.[3–5] Hence means are at hand for predicting whether any particular formulation will or will not yield a visual match.

However, it is generally recognized that the inverse problem of predicting directly from spectrophotometric data the formulation required to provide a match, has presented difficulties.[6]

In one specific application, namely, the color formulation of mixed wool fibers, Duntley[7] has proposed a method of formulating. However, this method is limited in that it is proposed for application only to the problem of fiber blending, and in that it is effective only when blending fibers which do not differ greatly in color.

In this paper a method is described which is applicable to all problems of color matching and which applies over entire ranges of absorption.

The development of the present method depends on the fact that it is generally possible to find a function such that for a mixture of colorants A, B, C in quantities a, b, c in a given substrate the reflectance or transmission R of the mixture is given by the equation

$$F(R)=aF(R_a)+bF(R_b)+cF(R_c), \qquad (1)$$

where R_a, R_b, R_c are the reflectances or transmissions of A, B, C. These additive functions are known for problems of major interest. Thus for specular transmission the function is[8]

$$F(R)=\log 1/R. \qquad (2)$$

Also, extensive tests, which it is proposed to report in greater detail in a subsequent paper, have shown that in a large class of color applications, involving especially the dyeing and pigmenting of fabrics and paper, the formula

$$F(R)=(1-b)^2/2b \qquad (3)$$

is closely applicable, where $b=R-s$ and s is a constant appropriate to the base to which the color has been applied, and which may be conceived of as representative of reflectance at the surface of the medium.

For mixtures of small particles, such as wool fibers, $F(R)$ has been shown to be capable of representation in the form[9]

$$F(R)=\frac{1-R}{m(R-0.01)+0.01}, \qquad (4)$$

where m = approximately 0.15 for wool mixtures.

For mixtures of independently absorbing elements the appropriate relation is merely $F(R)=R$. An example of this type is found in several color photographic processes referred to in general as screen plate process color reproduction.

With $F(R)$ known, the general method of formulation prediction is to make a preliminary

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* A contribution of the Research Department, Calco Chemical Division, American Cyanamid Company, Bound Brook, New Jersey.

[1] D. B. Judd, J. Opt. Soc. Am. **23**, 194, 359 (1933).

[2] A. C. Hardy, *Handbook of Colorimetry* (Technology Press, 1936).

[3] O. W. Pineo, U. S. Pat. No. 2,176,013 (1939), U. S. Pat. No. 2,218,357 (1940).

[4] S. Q. Duntley, J. Opt. Soc. Am. **32**, 61 (1942).

[5] J. L. Saunderson, J. Opt. Soc. Am. **32**, 727 (1942).

[6] J. G. Grundy, Proc. Phys. Soc. **54**, 1 (1942).

[7] S. Q. Duntley, Am. Dye Reporter **30**, 698 (1941).

[8] A. Beer, Ann. d. Physik (Leipzig) **86**, 78 (1852).

[9] E. I. Stearns and F. Noechel, to be published in Am. Dye Reporter.

formulation, and then to apply a process of successive approximation until within the tolerances.

Empirically it is found that a good preliminary formulation may be had by determining a, b, c from the equations.

$$\begin{aligned} a\sum_x F(R_a)+b\sum_x F(R_b)\\ +c\sum_x F(R_c)=\sum_x F(R_s),\\ a\sum_y F(R_a)+b\sum_y F(R_b)\\ +c\sum_y F(R_c)=\sum_y F(R_s),\\ a\sum_z F(R_a)+b\sum_z F(R_b)\\ +c\sum_z F(R_c)=\sum_z F(R_s), \end{aligned} \tag{5}$$

where $\sum_x$, $\sum_y$, $\sum_z$ are summations at suitably selected ordinates and R_s is the reflectance of the standard.

The foregoing Eqs. (5) provide a preliminary and approximate method of formulation. The further approach to an exact match may be gained by computing the small corrections which it is necessary to apply to the approximate formulation using for this purpose a formula based on the approximately linear relationship of small changes in composition to resultant small changes in reflectance. Thus sufficiently small changes Δa, Δb, Δc in a, b, c, produce a small change ΔR in R according to

$$\Delta R=\frac{\partial R}{\partial a}\Delta a+\frac{\partial R}{\partial b}\Delta b+\frac{\partial R}{\partial c}\Delta c. \tag{6}$$

Taking the partial derivative of Eq. (1)

$$\frac{\partial F(R)\partial R}{\partial R\partial a}=F(R_a), \tag{7}$$

whence

$$\frac{\partial R}{\partial a}=\frac{F(R_a)}{F'(R)}. \tag{8}$$

Similarly

$$\frac{\partial R}{\partial b}=\frac{F(R_b)}{F'(R)}, \qquad \frac{\partial R}{\partial c}=\frac{F(R_c)}{F'(R)}. \tag{9}$$

Introducing the nomenclature of reference 2 the change in tristimulus coefficient associated with a change in reflectance ΔR is

$$\Delta X=\int E\bar{x}\Delta R d\lambda. \tag{10}$$

Combining Eqs. (6), (8), (9), and (10)

$$\begin{aligned} \Delta X=\Delta a\int E\bar{x}\frac{F(R_a)}{F'(R)}d\lambda+\Delta b\int E\bar{x}\frac{F(R_b)}{F'(R)}d\lambda\\ +\Delta c\int E\bar{x}\frac{F(R_c)}{F'(R)}d\lambda. \end{aligned} \tag{11}$$

Similar relationships apply to ΔY and ΔZ. Then, taking ΔX, ΔY, and ΔZ as the differences between tristimulus values of the standard and the preliminary formulation, three equations are available which define all conditions necessary to a match subject to the degree of approximation of Eq. (6).

Where the results of a single calculation do not provide a good enough match any desired degree of improvement may be arrived at by a process of successive calculations. In practice it has been found a workable simplification in such successive calculations to use throughout the same values of $\int E\bar{x}F(R_a)d\lambda/F'(R)$, etc., computed in the first calculation. For determining whether or not the match is sufficiently close to the standard a color difference formula is needed.*

Six equations are introduced in sets of Eqs. (5) and (11) if it is desired to provide a match in two illuminants. Formulations calling for negative amounts of dye indicate the impossibility of a perfect match. No simple method has been worked out to determine in what proportion three dyes should be mixed to produce the closest visual match in cases where an exact match is impossible.

The method outlined has been found effective in practical problems.

It is desired to note that in the first form of Eq. (5) individual ordinates rather than a summation was used. This form was not wholly adequate. The suggestion of using a summation was due to C. F. Pinzka who also supervised calculations.

* The best presently available formula appears to be that of D. J. Judd, Am. J. Psychol. **52**, 418 (1939). See also R. S. Hunter, J. Opt. Soc. Am. **32**, 509 (1942).

Replacing a Thin Lens by a Thick Lens*

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(Received December 10, 1943)

IN the precalculation of optical systems, equations for thin lenses are frequently employed. By using theoretical equations, or by tracing finite rays through a system of thin lenses† with finite separations, the image errors can be corrected to a certain degree, but we must eventually replace the thin lenses by lenses having finite thicknesses.

Various procedures have been proposed for replacing a thin lens by an equivalent thick lens, and vice versa. It is obvious that the power of the lens should be kept constant, but there is left still one degree of freedom, and it is not obvious what second quantity should be specified. One such proposal, which involves an elaborate procedure, was made by Berek.[1]

Our solution to this problem is to keep the "bending" constant. That is, if we call the radii of the thick lens r_1 and r_2, and the radii of the corresponding thin lens R_1 and R_2,

$$r_2/r_1=R_2/R_1=\alpha. \tag{1}$$

We shall calculate the radii and the nodal points of a lens, as well as the Petzval sum, as functions of the thickness d, if the power Φ of the lens and the bending α are specified. We begin with the following well-known formulas. If n is the refractive index of the lens, the power is

$$\Phi=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)+\frac{d(n-1)^2}{n}\cdot\frac{1}{r_1}\cdot\frac{1}{r_2}$$
$$=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right), \tag{2}$$

the distances of the nodal points from their respective vertices are

$$s_n=-\frac{d}{n}\cdot\frac{1}{r_2}\cdot\frac{n-1}{\Phi},$$
$$s_n'=-\frac{d}{n}\cdot\frac{1}{r_1}\cdot\frac{n-1}{\Phi}; \tag{3}$$

and the Petzval sum P is given by

$$P=\frac{n-1}{n}\left(\frac{1}{r_1}-\frac{1}{r_2}\right). \tag{4}$$

We put α in Eq. (2), and find, for r_1 and r_2, the quadratic equations:

$$r_1^2\Phi-(n-1)\frac{\alpha-1}{\alpha}r_1-\frac{d(n-1)^2}{n\alpha}=0,$$
$$r_2^2\Phi-(n-1)(\alpha-1)r_2-\frac{d(n-1)^2}{n}\alpha=0, \tag{5}$$

or

$$r_1\Phi=\frac{n-1}{2}\left(\frac{\alpha-1}{\alpha}\right)\left\{1+\left[1+4\frac{d}{n}\cdot\frac{\alpha\Phi}{(\alpha-1)^2}\right]^{\frac{1}{2}}\right\},$$
$$r_2\Phi=\frac{n-1}{2}(\alpha-1)\left\{1+\left[1+4\frac{d}{n}\cdot\frac{\alpha\Phi}{(\alpha-1)^2}\right]^{\frac{1}{2}}\right\}; \tag{5a}$$

which becomes, if $d=0$,

$$r_1\Phi=(n-1)\left(\frac{\alpha-1}{\alpha}\right),$$
$$r_2\Phi=(n-1)(\alpha-1); \tag{5b}$$

or, finally, for the change of radius with thickness,

$$(r_1-R_1)\Phi=$$
$$\frac{n-1}{2}\left(\frac{1-\alpha}{\alpha}\right)\left\{1-\left[1+4\frac{d}{n}\cdot\frac{\alpha\Phi}{(\alpha-1)^2}\right]^{\frac{1}{2}}\right\},$$
$$(r_2-R_2)\Phi=$$
$$\frac{n-1}{2}(1-\alpha)\left\{1-\left[1+4\frac{d}{n}\cdot\frac{\alpha\Phi}{(\alpha-1)^2}\right]^{\frac{1}{2}}\right\}. \tag{6}$$

---

* Communication No. 958 from the Kodak Research Laboratories.

† Although such a lens, especially a convex lens having zero center thickness and finite aperture, is purely hypothetical, finite rays may be traced through it, since the tracing formulas work equally well whether the edge thickness of a lens is positive, indicating actual existence, or negative, indicating apertures beyond the intersection of the surfaces. Such a trace serves as a good approximation to a trace with finite thicknesses.

[1] M. Berek, *Grundlagen der praktische Optik, Analyse und Synthese optischen Systeme* (W. de Gruyter & Co., Berlin, 1930), p. 86.

Similarly, we find for the positions of the nodal points:

$$s_n\Phi=\frac{1}{2}\cdot\frac{\alpha-1}{\alpha}\left\{1-\left[1+4\frac{d}{n}\cdot\frac{\alpha\Phi}{(\alpha-1)^2}\right]^{\frac{1}{2}}\right\},$$
$$s_n'\Phi=\frac{1}{2}(\alpha-1)\left\{1-\left[1+4\frac{d}{n}\cdot\frac{\alpha\Phi}{(\alpha-1)^2}\right]^{\frac{1}{2}}\right\}; \tag{7}$$

and for the Petzval sum:

$$P=-\frac{(\alpha-1)^2}{2d\alpha}\Phi\left\{1-\left[1+4\frac{d}{n}\cdot\frac{\alpha\Phi}{(\alpha-1)^2}\right]^{\frac{1}{2}}\right\}. \tag{8}$$

Let us write, for abbreviation,

$$1-\left[1+4\frac{d}{n}\cdot\frac{\alpha\Phi}{(\alpha-1)^2}\right]^{\frac{1}{2}}=g(d). \tag{9}$$

Equations (6), (7), and (8) then become

$$(r_1-R_1)\Phi=\frac{n-1}{2}\left(\frac{1-\alpha}{\alpha}\right)g(d),$$
$$(r_2-R_2)\Phi=\frac{n-1}{2}(1-\alpha)g(d),$$
$$s_n\Phi=\frac{1}{2}\cdot\frac{\alpha-1}{\alpha}g(d), \tag{10}$$
$$s_n'\Phi=\tfrac{1}{2}(\alpha-1)g(d),$$
$$P=-\frac{(\alpha-1)^2}{2d\alpha}g(d).$$

If we assume the thickness to be small, so that the square and higher powers of d may be neglected, we find the following approximation formulas, which are sufficient for most practical purposes:

$$g(d)=-2\frac{d}{n}\cdot\frac{\alpha\Phi}{(\alpha-1)^2};$$

and

$$r_1-R_1=[(n-1)/(\alpha-1)](d/n),$$
$$r_2-R_2=[(n-1)/(\alpha-1)](d/n)\alpha,$$
$$s_n=-(d/n)[1/(\alpha-1)], \tag{11}$$
$$s_n'=-(d/n)[\alpha/(\alpha-1)],$$
$$P=\Phi/n.$$

We see that the Petzval sum is unchanged if the higher powers of the thickness are neglected.

When a system of thin lenses is replaced by a system of thick lenses, we must of course take care that the separations between successive lenses are measured from nodal point to nodal point. Thus, the distances of the nodal points from the vertices (s_n and s_n') must be added to the actual vertex distance between lenses.

This method of replacing a thin lens by a thick one leaves the Gaussian optics, and therefore the color correction, of the system intact; and, in first approximation, the Petzval sum also is unchanged.

Accessory Equipment for Spectrochemical Analysis*

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INTRODUCTION

THIS paper describes miscellaneous accessory equipment used in this laboratory for the spectrochemical analysis of magnesium alloys. Consideration of the purpose of the equipment here described shows that rigid control of the experimental conditions during the analysis as well as convenience and efficiency of operation dictates the design of equipment. In the relatively few cases where convenience is not compatible with duplication of conditions, the latter must be favored. Much of the equipment in use in this laboratory on the routine analytical work has been described by others. The spectra are photographed with a Bausch and Lomb Medium Quartz spectrograph, the spark source follows the design of Feussner,[1] the arc is similar to that of Duffendack and Wolf,[2] the microphotometer is of the Michigan design,[3] and the calculating board[4] is of the conventional type. Apparatus for controlling the development and drying of the plates is similar in principle to that now commercially available.

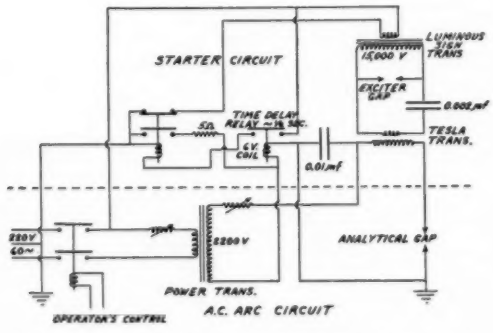

FIG. 1. Automatic arc starter. The starter is shown above the dotted line, connected to a conventional arc source below.

The equipment described below may best be termed "accessory," in the sense that its use is not necessarily essential, but rather that it offers greater convenience of operation and control of conditions. The value of such equipment is probably best emphasized by the fact that during the last two years the analytical load carried by this laboratory has increased by a factor of 300 percent, while the personnel engaged in the routine work has increased only 50 percent. The "accessory" equipment has been an important factor in the increased efficiency and accuracy of the routine spectrochemical analysis of magnesium alloys.

AUTOMATIC ARC STARTER

Figure 1 shows a wiring diagram of a conventional high voltage a.c. arc equipped with an automatic arc starter. Electrical ignition systems have been described by Pfeilsticker[5] and others,[6] and the starter part of Fig. 1 is not essentially different from these. A luminous sign transformer of standard rating, 15,000 v at 240 v-amp., is used to power the exciter circuit. When the contactor in the primary of the arc power transformer is closed, the starter circuit is excited simultaneously. Coupling between the starter circuit and the arc secondary circuit is obtained by means of a Tesla type transformer. When the starter circuit is excited, a high frequency oscillatory discharge takes place between the electrodes of the analytical gap. The arc current will follow the starter current across the gap, and when the electrodes become heated sufficiently the arc will be self-maintaining. For magnesium alloy electrodes, the arc will maintain itself after a tenth of a second. For iron electrodes, the starter must be run several seconds. The coil of the time delay relay is connected to the low

* Paper presented at the Twenty-Eighth Annual Meeting of the Optical Society of America, Pittsburgh, Pennsylvania, October 7–9, 1943.

[1] O. Feussner, Arch. Eisenhüttenwes **6**, 551 (1932–33); also O. Feussner, U. S. Patent 1,971,215. See also H. B. Vincent and R. A. Sawyer, J. App. Phys. **8**, 163 (1937).

[2] Duffendack and Thompson, Proc. A. S. T. M. **36**, Part II (1936).

[3] H. B. Vincent and R. A. Sawyer, J. Opt. Soc. Am. **31**, 639 (1941).

[4] J. S. Owens, Metals and Alloys (January, 1938).

[5] K. Z. Pfeilsticker, Zeits. f. Elektrochemie **43**, 719 (1937).

[6] H. Hemmendinger, J. Opt. Soc. Am. **32**, 149 (1942); F. G. Brockman and F. P. Hochgesang, Ind. Eng. Chem. Anal. Ed. **14**, 796 (1942).

voltage side of the secondary of the arc power transformer, so that the arc current flows through the coil. When the arc starts, the time delay relay closes after the delay period. The second relay is then activated, opening the primary of the exciter circuit and introducing a five-ohm power resistor in parallel with the time delay relay coil. The starter thus goes off automatically after about one-half second, and the time delay relay remains closed as long as the arc continues to run. If the arc should go out, the starter comes on immediately until the arc is re-ignited. When using electrodes for which the arc is more difficult to ignite than for magnesium alloy, a longer delay period should be used.

A time delay relay with normally closed contacts could be used, and if the contacts had a rating sufficient to carry the current for the luminous sign transformer, the second relay would not be necessary. The five-ohm resistor was inserted in parallel with the coil of the time delay relay in order to reduce the current through the coil after the relay has closed. This procedure was considered advisable in this case because the particular time delay relay used required a much larger current to close the relay than was necessary to keep it closed. The Tesla type transformer was constructed from two concentric solenoids. The outer coil contained 30 turns of number 24 wire wound with a 4-inch diameter, and the inner coil had a diameter of $3\frac{1}{2}$ inches and consisted of 75 turns of number 12 wire; both coils had $\frac{1}{8}$" spacing between turns. The 0.01-μf filter condenser is sufficient to prevent any high voltage from damaging the power transformer. It should have sufficient insulation to withstand 5000 volts. The condenser in the exciter circuit was tested at 15,000 volts. Both condensers were made by piling up alternate sheets of copper and glass in a pan containing molten wax.

STABILIZED MICROPHOTOMETER POWER SUPPLY

The power supply here described is one which has been used with considerable success for several years. It is simple in design, gives trouble-free service, and is easily adaptable to any microphotometer requiring power for a lamp at low voltage and large current. The principles involved in the circuit are not new, and a description is justified here only by the belief that many workers engaged in spectrochemical analysis would welcome publication of a complete unit. The system was first suggested to the authors by Dr. J. L. Lawson at the University of Michigan.

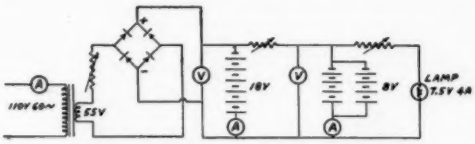

FIG. 2. Stabilized microphotometer power supply. Adjustable resistors from left to right are R_1, R_2, R_3.

The unit is stabilized by means of the well-known "floating" battery principle (Fig. 2). The 110-volt 60-cycle current is reduced in voltage by means of the transformer and R_1 and rectified so that the d.c. voltage across the first battery is just sufficient to maintain a small charging current. The voltage is reduced at the second battery by the resistor R_2, and again from the second battery voltage to the voltage supplied the lamp by the resistor R_3. The stabilization against line voltage fluctuations is much better when two sets of batteries are used at two voltages, as shown, than when the same number of batteries are all used at the same voltage. Ordinary 6-volt automobile storage batteries have been used in this laboratory. The first set of batteries consists of as many batteries in series as are required to obtain the necessary voltage, while the second set contains two sets of batteries arranged in parallel for increased capacity for stabilization. A charging current of about 0.1 amp. through each battery has been found to be satisfactory. The second ammeter, therefore, shows about 0.2 amp.

The resistors should be of the sliding contact type which is clamped in place rather than the rheostat type which does not make sufficiently good contact for this purpose. The additional convenience of the latter type is not necessary since the unit should require little or no adjustment after it has been set up properly for operation. The service required by the unit is limited to checking the water in the batteries every six weeks.

It was the original intention when this unit was constructed to substitute a current regulator tube for the resistance R_1 to minimize slow drift

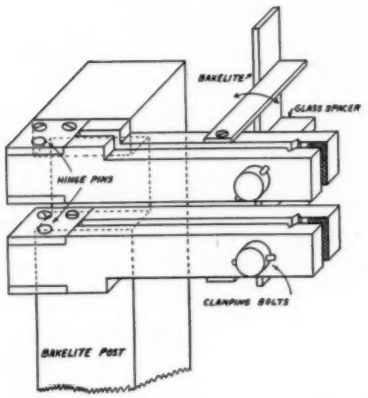

FIG. 3. Electrode clamps particularly suitable for magnesium alloys. The electrode spacing mechanism can be swung forward so that the glass spacer is between the two clamps.

effects. The system has been so satisfactory, however, that this was never done. Within twenty minutes, the average time required to measure the line intensities on a plate, the drift in transparency of the clear plate rarely exceeds 2 mm and usually is less than 1 mm on a 500-mm scale.

ELECTRODE CLAMPS

The electrode clamps shown in Fig. 3 are designed to hold rectangular or cylindrical electrodes very tightly and always in exactly the same position. The clamps are heavy enough to carry safely the currents encountered with the arcs and sparks used in spectrochemical work The clamps require no artificial cooling when used with a spark, but cooling may be desirable for continuous use with an arc. Artificial cooling can be accomplished either by soldering a small copper pipe on the outside of each of the jaws, or by drilling a hole inside the jaws to allow the circulation of water or air. Several sets of clamps of this design have been constructed for use in routine work. The advantages of the design include strength of construction, rigidity and lack of adjustable parts, large thermal capacity, positive pressure for gripping electrodes, and accurate alignment of electrodes. An experienced operator can load the clamps with two electrodes in 10 seconds.

The clamp jaws are constructed of $\frac{1}{2}''$ steel. The two jaws next to the Bakelite post are bolted to the post, electrical connections being made through the bolts. The front jaws are held by the hinge pins, which allow the jaws to rotate enough to get electrodes in and out. The clamping bolts pass freely through the front jaws, and are threaded into the rear jaws. A removable key or wrench is provided to tighten the bolts. The electrode spacing mechanism swings parallel to the jaws to space the electrodes, and can be swung out of the way for the discharge. The V-groove is intended to be narrow enough in comparison to the diameter of the electrodes so that the edges grip the electrodes firmly when the clamps are tightened.

The lack of adjustments on the clamps is considered an advantage for routine work. Once proper alignment with respect to the spectrograph has been found, the clamps require no further adjustment.

ELECTRODE SHAPER

A simple and straightforward design was followed in constructing the apparatus used in this laboratory for shaping the ends of cylindrical electrodes in preparation for the discharge. A $\frac{1}{4}$-hp motor with built-in automatic brake was fitted with a Jacobs keyless chuck mounted on the motor shaft. The motor was mounted on an iron base together with a compound tool rest fitted with a cutting bit. A three-phase motor was

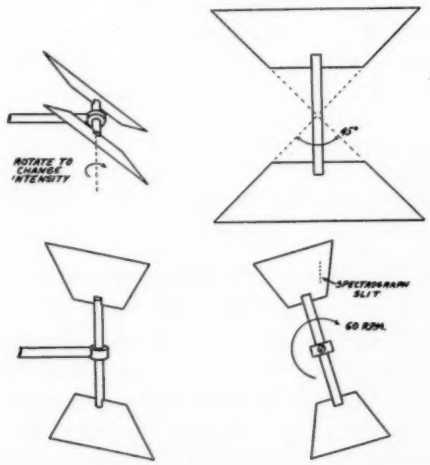

FIG. 4. Intensity control sector, having a continuously variable transmission from 50 percent to 100 percent.

used because of the frequent starting required in routine operation. The cutting operation is identical to that of a lathe, but the frequent starting and stopping can be done more conveniently than with the usual lathe. The motor starts nearly instantaneously, and stops in about one second. Construction of the unit is simplified in that no bearing surfaces are required. The apparatus has been in use for about one year without showing any noticeable effect of wear. Mr. W. M. Weddell was partly responsible for the design and construction of this apparatus.

INTENSITY CONTROL VANE

The vane shown in Fig. 4 was constructed to provide a continuously variable control of the intensity of the photographed spectra. The vane is used to maintain the density of a standard line at some constant value when a difference in the speed of the photographic plates occurs. The vane has a continuously variable transmission between 50 percent and 100 percent, and affords a method of intensity control which does not depend upon changing source distance, exposure time, or the use of filters and screens. The vane is rotated at 1 revolution per second to avoid any possibility of a stroboscopic effect when used with the interrupted spark. Although calibration was carried out under the actual operating conditions using the vane in order to nullify any possible effect from the intermittancy of the exposure, no evidence for any intermittancy effect in this case has yet been discovered.

A layout of the vane is shown in the upper right-hand corner of Fig. 4. To adjust the transmission factor, the vane can be rotated in the collar mounted on the end of the shaft connected to the 60-r.p.m. motor. The apparatus should be used such that the shaft to the motor is parallel to but about two inches lower than the optical axis of the spectrograph.

AUTOMATIC EXPOSURE APPARATUS

A time-saving system has been constructed to minimize the labor required to operate the spectrograph used on the routine analysis of magnesium alloys. The actual wiring diagram of the system appears to be so closely connected with the particular requirements of this laboratory that a detailed description would not be justified. The general operation of the system can best be followed by reference to the block diagram of Fig. 5. Briefly, the timing system will make a spark exposure followed by an arc exposure, and move the plate down the proper distance after each. It has been found desirable to obtain both spark and arc exposures on magnesium alloys to cover adequately the analytical range required.

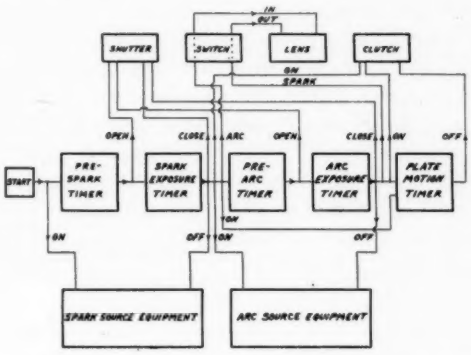

FIG. 5. Block diagram showing operating functions of the timing system for automatic spectrograph.

After the operator has prepared two electrodes and has loaded them into the clamps, he pushes a "start" button, and then returns to prepare two more electrodes from the next sample to be analyzed. The spark discharge begins immediately, as well as the first timer. A magnetic shutter is closed during the first part of the spark discharge, and is opened when the first timer reaches the time interval for which it is set. The second timer starts when the shutter opens, and regulates the length of the spark exposure. At the end of the spark exposure, the spark goes off, the shutter closes, a high voltage relay changes the leads from the electrode clamps over to the arc, a lens comes into position for the arc exposure, the arc is turned on and ignited by the automatic starter, the clutch mechanism controlling the plate motion is activated, and both the arc pre-exposure timer and the plate motion timer are started. After several seconds, the plate motion timer releases the clutch mechanism and the arc pre-exposure timer opens the shutter and starts the arc exposure timer. After the arc exposure, the arc is turned off, the high voltage

relay goes back to the spark position, the lens is removed, the shutter closes, the plate motion clutch is again activated, and the plate motion timer starts. When the plate has been moved down the correct distance, the timer again releases the clutch. Thus the system is entirely reset ready to repeat the cycle on the next sample. Provision is also made with appropriate switches to permit running only sparks or only arcs. The complete cycle, under the present operating conditions, requires one minute and 20 seconds. An operator is expected to prepare the next set of electrodes within this interval.

The timer circuit was constructed from five General Electric Type TSA-10 time switches, several auxiliary relays and solenoids, and a magnetic clutch with a driving motor. The specially constructed parts included the high voltage relay, the lens mechanism, and the magnetic shutter.

ISCC Meeting—March 1, 1944

THE 13th Annual Meeting of the Inter-Society Color Council will be held at the Hotel Pennsylvania, New York City, on March 1, 1944, the day prior to meetings of the Optical Society of America.

The morning session, to be devoted to a discussion of small color differences, will be cosponsored by the American Association of Textile Chemists & Colorists and by the Federation of Paint and Varnish Production Clubs. It is especially appropriate that this session be cosponsored by these two member groups of the ISCC since they represent textile and paint interests particularly interested in practical problems involving small color differences.

The afternoon session will consist of a discussion of more general topics, and a business session at which annual reports from each member body, annual reports of officers, and of committees will be presented.

All persons interested are invited to attend.